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TERRITORIAL INDUSTRIAL COMPLEXES: OPTIMISATION MODELS AND GENERAL ASPECTS

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Translated from the Russian by *H. Campbell Creighton*

ФОРМИРОВАНИЕ ТЕРРИТОРИАЛЬНО-ПРОИЗВОДСТВЕННЫХ
КОМПЛЕКСОВ: ОБЩИЕ ВОПРОСЫ И МОДЕЛИ ОПТИМИЗАЦИИ

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FOREWORD

One of the many research teams and groups in the USSR engaged in building mathematical economic models and systems of models for long-term territorial planning is the Institute of Economics and Industrial Organisation of the Siberian Department of the USSR Academy of Sciences. Its basic propositions on employing mathematical economic methods of modelling in territorial industrial planning, and its variant of a system of models have been set out in group monographs and individual papers prepared by its staff (e. g. Optimal Territorial Industrial Planning, Nauka Publishers, Novosibirsk, 1969; A. G. Aganbegian, K. A. Bagrinovsky, and A. G. Granberg, A System of Models for Economic Planning, Mysl Publishers, Moscow, 1972, and others).

The main object of the work of the Institute's sector on the forming of territorial industrial complexes is to develop a logical scheme and instrument for optimising their formation. The main subjects being studied are large intraregional territorial industrial complexes of the rank of the Sayan, Central Ob, Central Angara (Bratsk-Ilimsk), Irkutsk-Cheremkhovo, and Kuzbas complexes, which are approximately of the same size, and of equal national significance. Complexes of other ranking, viz. economic regions and industrial centres, are investigated in other studies being made in the Institute. Our sector concerns itself with them only insofar *as is necessary to analyse the formation and functioning of the objects of our research.*

The main objectives of this collection of papers are the following:

(a) study of territorial industrial complexes as an object of modelling;

(b) the distinguishing of factors determining the time, rates, scale, and direction of the development both of territorial industrial complexes as a whole and of their separate parts, the siting of all the economic elements of a complex, and the shaping of its external and internal links;

(c) the means of quantifying these factors and conditions;

(d) the search for means for adequately expressing the main features of a complex mathematically as a form of spatial organisation of the productive forces, of the factors affecting the formation of complexes and their operation conditions;

(e) the development of approaches to the use of mathematical economic modelling for resolving territorial pre-planning problems;

(f) the development of an instrument, i.e. of a group of models, for optimising the formation of a territorial industrial complex as a whole and of its separate elements;

(g) determination of the composition of the initial information and of the specific features of preparing the data for solving mathematical economic problems relating to complexes;

(h) the working out of directions and apparatus for analysing the results of the solutions of these optimisation problems concerning the forming of a complex, and the finding of ways of applying the results in practice in pre-planning territorial research.

Several stages can be distinguished in our formulation of the approach and instrument for solving the optimisation problems of organising territorial industrial complexes. The beginning of the work (1964-5) was an investigation of a sepa-

rate complex by means of one territorial model. Then (1965-6) we went on to the investigation of a system of complexes and the building of models of production cycles. The next stage (1966-8) was a transition to breaking the problems for a system of complexes and individual ones down into sub-problems, and the development of models of various kinds for them. That was a qualitatively new period, differing in principle from the preceding ones, the stage of delineating a sub-system of production models of a region and of a group of models for optimising development of the industries of a single territorial industrial complex, and of arriving at a solution of the first practical problems. By 1970 quite clear ideas of the object, of the content of the problems, and of the possibilities of the models and programs had taken shape. Finally, the next four years (1971-4) were a period of the thinking out and checking of the approach and models by tackling practical problems of large dimension, and by exploring certain matters of principle in depth, though still without resolving them, such as improving analysis of the results of solutions, passing to the use of dynamic models, and investigation of the links between the models themselves and the other elements of the system of models for long-term territorial planning.

The directions of the work also determined the structure of this book. The first two parts are devoted to an exposition of the general problems of modelling and to a detailed description of the concrete models. The third part presents the results of certain exploratory work. Two chapters (11 and 12) are devoted to analysing the properties of the territorial industrial regional mesomodels suggested by us, and to describing their main features. An important trend in our future work will be to improve analysis of the results obtained; for that reason we investigate shadow prices (Chapter 13) and the indeterminacy field of optimal solutions (Chapter 14). Since we still do not have working non-linear programs, it is urgent to explore the ways of taking non-linear dependences into account in *linear models (Chapter 15). Finally, we have tried to determine the effect of the solution of territorial problems.*

Our book includes only those models that can be used to tackle practical problems.

At the beginning of the work in our Institute on the optimisation of formation of territorial industrial complexes neither the object of the research itself nor the problems of organising it were new. We adopted the commonly accepted propositions of the theory of territorial industrial complexes as a form of territorial division of labour, and set ourselves a very narrow aim, that of trying to find the application of a new apparatus to known problems. Our team tried to make do with the terms already in currency; taking the practical bent of the work into account, we oriented ourselves simply on the initial information available for processing and already adjusted programs. When the use of individual terms created ambiguity, we introduced our own understanding of them. The terms 'space', and 'territory' are employed as synonyms to avoid combinations like 'territorial structure of a territorial industrial complex'. Three terms are used to signify territorial industrial taxonomic units, viz. territorial industrial combination, irrespective of its rank; industrial centre, and territorial industrial complex, when it is a matter of the objects that we are investigating.

On the whole, as the work proceeded, we increasingly came to the view that the flexibility of the modelling process made it possible to represent the main features of a complex as forms of the territorial organisation of the productive forces and patterns of the organisation of complexes while the analytical possibilities of models made it possible to understand the inner mechanism of the process itself. Going over to quantitative methods helped us not only to improve the standard of our research, making it more rigorous and convincing, but also to improve the practical return from it.

We consider the approach and the group of models for optimising the formation of complexes set out in our papers as a pro-

posal for a logical scheme and instrument for dealing with certain problems of long-term territorial planning at the pre-planning, investigatory stage. No modelling, however perfect, guarantees full representation of all the processes in the forming and functioning of complex economic elements, including territorial industrial complexes. We, therefore, consider the variant of the formation of a complex obtained through optimisation as only a certain approximation to the economic optimum for a certain set of given parameters and permissible limits of indeterminacy.

The approach proposed, and the sequence of the solutions and models, are not universal. They were developed in application to Siberia and reflect the conditions of the formation and functioning of territorial industrial complexes in newly developed areas.

M. K. Bandman

Part I INITIAL PROPOSITIONS FOR OPTIMISING THE FORMATION OF TERRITORIAL INDUSTRIAL COMPLEXES

1

THE SIGNIFICANCE, SEQUENCE, AND INSTRUMENT OF OPTIMISING THE FORMATION OF TERRITORIAL INDUSTRIAL COMPLEXES

M. K. Bandman

1. PROBLEMS OF IMPROVING THE PRE-PLANNING PREPARATION OF TERRITORIAL INDUSTRIAL COMPLEXES

Territorial organisation of the productive forces, based on maximum employment of the natural and economic conditions of separate areas, joint location of all the elements of the economy, and comprehensive exploitation of resources and of the advantages of the territorial division of labour are one way of raising the efficiency of social production. The effectiveness of a territorial industrial complex, compared with scattered distribution of undertakings or their simple grouping, is expressed in a saving of natural resources and labour, reduction of capital investment, lowering of current outlays, reduction of the volume of transport work, speeding up of the rate of turnover of circulating assets, saving of means on the building and functioning of the infrastructure.

The territorial organisation of industry is especially important in areas that are being newly opened up and developed. A striking example of that is Siberia. Because of the specific character of its natural and economic conditions, development of Siberia's productive forces is taking the path of selective exploitation of resources, of focal development of territory, and the forming of separate territorial industrial combinations differing in scale, significance, and structure, their agglomerations and whole systems. The most progressive and promising forms of the spatial organisation of industry in Siberian conditions have proved to be industrial centres, and especially territorial industrial complexes.

Experience of developing the productive forces of Siberia has confirmed the correctness of the direction chosen for the territorial organisation of industry. By concentrating investment in limited areas, it has been possible to build the biggest power centres in the USSR, and foci of power-intensive branches of the non-ferrous metal and chemical industries, and of lumbering and paper-making, with relatively moderate outlays and in a relatively short period of time. In that way the foundations have been laid for future large-scale territorial industrial complexes, and the premises created for converting Siberia into the power base of the Soviet Union and the main area for the production of many non-ferrous metals, and of products of the chemical, lumbering, and other industries.

Large territorial industrial complexes as objects of long-term planning. The objects of Soviet long-term planning at the present time are the country as a whole, Union republics, administrative territories, regions, and autonomous national units, cities, and the bottom-level administrative districts. Plans are compiled for them, and there are agencies responsible for implementing these plans. Many contemporary economic problems, however, especially long-term ones, do not fall within the confines of one administrative unit but embrace several adjoining regions, or part of the area of a region or territory. In that case the objective basis is laid for organising major economic regions (such as the Ob-Irtysh, Angara-Yenisei, or Central Siberian) and large territorial industrial complexes. In Siberian conditions big complexes seldom coincide territorially with today's administrative units, but are often part of a territory or region.

In the current practice of long-term planning, economic regions are treated simply as the objects of plan analysis, but neither they nor, even more, complexes are the objects of long-term planning. Yet, both by the scale of the investment programmes and the significance of the problems to be tackled Siberian complexes are equal in importance to many of the regions and autonomous republics for which long-term plans are compiled. Because complexes are not treated as independent objects of long-term territorial planning, development of the individual elements of their economies is planned in isolation, which leads to lack of co-ordination of their formation and loss of their most important advantages as forms of the territorial organisation of industry, viz. balanced development. Consequently, (1) *Large-scale complexes need to be made the object of comprehensive pre-planning research*, in which the principles of the general schemes should be drafted for every projected complex, and above all, for every complex being organised, a general plan of the development and allocation of the productive forces. (2) For priority complexes, i.e. for ones already being formed, *not only should general schemes be drafted, but also long-period and five-year plans* embracing all projects irrespective of their departmental subordination. (3) These *plans should be components of the long-term plan of the corresponding administrative unit* (*territory*, *region*), or included in the plans of Union republics or of the USSR itself as an independent construction project. That, as experience has shown, raises responsibility and eases control over plan fulfilment.*

The first steps have been taken in this direction. The 24th Congress of the CPSU introduced the concept of territorial industrial complexes as a form of organisation of the economy of areas within which important national economic tasks are being tackled. Some of the complexes singled out in the 1971-5 state economic development plan were the following: the Kuzbas, Bratsk-Ust Ilimsk, Irkutsk-Cheremkhovo, South Tajikistan, and Pavlodar-Ekibastuz complexes.

The general plan of a complex as a basic pre-planning document. The main pre-planning forecasting document for a complex should be the general scheme for the development and allocation of its productive forces, in which variants of the process of organising its whole economy, and its separate elements, are considered in both time and space. The general plan is the basis on the one hand for drafting long-term (general) and five-year plans, and on the other hand is an obligatory initial document for the long-term research of all the organisations whose interests are involved with the characteristics of a complex. Both these purposes of the general plan are exceptionally important because only in that way can the continuity of plans and unity of the initial positions be guaranteed in the pre-planning research of the many interested bodies.

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* In Soviet practice of drafting long-term plans there have been cases when major industrial and territorial problems have been singled out as independent sections of the plans. Examples are the Ural-Kuznetsk inter-regional problem and the industrial combine at the Dnieper Hydroelectric Power Station.

In the general plan for the development and allocation of the productive forces of the USSR drafted for the period up to 1980 the matters considered included problems of the development and location of the sectors of the economy and of the productive forces of Union republics and economic regions, and special plans for very large economic complexes.

The plan was adopted as an official document; the resolution of the Central Committee of the CPSU and of the USSR Council of Ministers of 28 May 1969 stated that from 1971 onward decisions on the planning and building of enterprises and undertakings should be taken in conformity with this plan.

The plan of a territorial industrial complex is drawn up on the basis of a comprehensive analysis of the initial conceptions for building this complex, and of forecasts of the trends in exploiting its resources, of its industrial structure, of the time and rates required for building the complex as a whole and the separate elements of its economy, of the specific engineering and planning conditions, systems of settlement, and forms of spatial organisation of its internal economy. The efficiency of future complexes, the building of which will call for heavy capital investment and several five-year periods of work largely depends on proper solution of the aggregate of these questions.

The drafting of such important generalising documents as the general plan for organising the individual elements of the territorial system of the USSR calls not only for the solution of many theoretical and organisational matters but also for improving the methods and instruments of the analysis itself. One way of raising the standard of pre-planning territorial research, in our view, is the gradual introduction of mathematical economic modelling methods. Their great advantage is the opportunity they present of simultaneously analysing a set of variants of very different situations in the forming and functioning both of individual complexes and of systems of complexes.

2. TERRITORIAL INDUSTRIAL COMPLEXES AS THE OBJECT OF MODELLING

A territorial industrial complex as a specific form of spatial organisation of the productive forces. The basis of the theory of the territorial organisation of the productive forces of socialist society was laid in the works of the founders of Marxism-Leninism. The first practical embodiments of the idea of the concentration and combining of industry, comprehensive development of resources and development of industry were the GOELRO Plan, the scheme of economic zoning of the USSR in the early 1920s, and the projects for the Ural-Kuznetsk Combine, the Dnieper Hydroelectric Power Station, the Great Volga and Angara-Yenisei schemes, and other plans.

Deepening of the theory, and analysis of the practice, of the spatial organisation of industry led to a theory of territorial industrial combinations. The origin of this theory is associated with the names of G. M. Krzhizhanovsky and I. G. Alexandrov. N. N. Kolosovsky also played a leading role in shaping this theory. Just as N. N. Baransky gave the first description of the Soviet economy according to the State Planning Committee's (Gosplan) economic regions, so Kolosovsky succinctly formulated the main initial propositions about industrial territorial combinations (complexes) and attempted to embody them in the practice of economic zoning and comprehensive pre-planning territorial research, 'reinforced the technological and economic aspects of Baransky's school and introduced the spirit and style of constructive solutions and computations into it'.* Striking examples of this research were Kolosovsky's work on the Ural-Kuznetsk Combine problem, transport construction, and, finally, matters of the territorial organisation of the productive forces of Eastern Siberia.*

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* Yu. G. Saushkin, *Ekonomicheskaya geografiya: istoriya, teoriya, metody i praktika* (Economic Geography: History, Theory, Methods, and Practice), Mysl Publishers, Moscow, 1973, pp. 116-17.

In subsequent years the ideas of industrial territorial combinations (complexes) have been developed further by pupils of Baransky and Kolosovsky (lecturers and graduates of the chairs of economic geography of the USSR of Moscow University), and by geographers and economists in Moscow, Kiev, Lvov, Sverdlovsk, Leningrad, and other cities.

The process of the development of this teaching, the role of the works of the founders of Marxism-Leninism, Lenin above all, and of individual scholars and schools, and the theory and practice of organising territorial systems have been widely discussed in the Soviet literature.

Our views on territorial industrial complexes, and our definition of their place in long-term territorial planning, are based on the ideas of I. G. Alexandrov, N. N. Baransky, A. E. Probst, Yu. G. Saushkin, I. M. Mayergoiz, and M. I. Pomus, and primarily on the propositions about industrial territorial combinations (complexes) expressed by N. N. Kolosovsky in his many works on economic zoning and territorial organisation of the productive forces.**

By a large *territorial industrial complex* we mean, in this book, *a planned, proportionately developing aggregate of stably interconnected units of the sectors of the economy* (industry, agriculture, construction, transport, and the non-productive sphere), *which have been built for joint solution of one or more major economic problems, and are concentrated in a relatively limited, compact* (undivided) *area*, possessing resources of a composition and scale adequate for the complex to be involved in solving these major problems, and which *ensure efficient use* (from the angle of the Soviet economy) *of local and outside resources, conservation of the environment, and reproduction of natural resources, and served by a single infrastructure.*

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* N. N. Kolosovsky, *Problemy territorialnoi organizatsii proizvoditelnykh sil Vostochnoi Sibiri* (Problems of the Territorial Organisa, tion of the Productive Forces of Eastern Siberia), Nauka Publishers-Novosibirsk, 1971, p. 176.

** N. N. Kolosovsky, *Osnovy ekonomicheskogo raionirovaniya* (Fundamentals of Economic Zoning), Gospolitizdat, Moscow, 1958, p. 200; *Teoriya ekonomicheskogo raionirovaniya* (Theory of Economic Zoning), Mysl Publishers, Moscow, 1969, p. 336.

Complexes must thus be *created* in accordance with a previously developed single plan of formation. "One must not call the simple coexistence of enterprises a complex; it should rather be called a grouping," Kolosovsky said.* It is not sufficient for the interconnection of facilities to be based simply on joint exploitation of local resources and use of the local labour force and infrastructure; a *sine qua non* of a complex is the *existence of interconnected industries that form the basis of the complex and its specialisation.*

The basis of a complex can be not only manufacturing industry or industry in general but also agriculture. What is important is not what industries underlie it but how they are *interconnected*, how the whole economy of the area is organised, and what role the industries play in the division of labour.

The time, rates, and directions of the development of a complex, and its rank, i.e. its position in the territorial industrial system of the whole country, depend above all on the *significance of the economic problem* that the area is involved in solving (generally because of its resources, less often because of its economico-geographical location).

Not every area can be the basis for building a complex. It is necessary, in order to organise a complex of a certain rank, on the one hand for the area to have the appropriate economic or natural potential, and on the other hand for there to be the possibility of controlling organisation of the complex through agencies of the appropriate rank.

The object and character of a complex are determined not by exploitation of all the area's resources but only by exploitation of certain of them, and by the scale and direction of their exploitation, which must be efficient not only and not so much from the standpoint of the area itself as from the angle of the economy as a whole.

It is important that the interconnected industries should actively promote economic development in the area, and jointly create and use the infrastructure.

Only in such circumstances can the maximum economic and social effect be obtained from complexes as the most progressive form of territorial organisation of the productive forces.

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* N. N. Kolosovsky, *Theory of Economic Zoning*, p. 142.

The effect of a territorial industrial complex, unlike sectoral systems, is built up not only as the result "of the fullest and most correct exploitation of natural and secondary raw materials at all stages of their processing, of reduction of transportation costs and of stockpiling and other losses in handling the raw material or product during the production process, of more synchronous working of undertakings, making it possible to use their circulating and fixed assets rationally, and so on",* but also through rational development of the infrastructure, exploitation of local resources (land and water) and use of labour force, and the organisation of management.

Kolosovsky considered an industrial territorial combination to be "social human labour organised in certain technical forms",** at the same time, however, he noted that by the term "industrial territorial combination" "must be meant a reciprocally conditioned (subordinated) aggregate of production facilities and settlement (inhabited places)".***

Thus, an industrial territorial combination, while having industry as its basis, also includes settlements, and that means their infrastructure and, of course, their population. In our view, as will be seen from the definition given above, a territorial industrial complex, unlike power-industrial cycles, territorial sectoral sub-systems, multistage systems of production, or any other such systems of production, be it also territorial, but generally sectoral or intersectoral, is a form of organisation of the whole economy of a certain area, one of the elements of the territorial industrial structure of the country.

Thus, in our view, a complex is a territorial industrial system in which elements of three major systems interact, viz. economic, demographic, and natural, the functioning of each of which is governed by its own objective laws and is studied by the appropriate sciences. A complex is organised and functions through the interaction of its constituent elements in conditions of a definite area and interval of time. We are, therefore, interested not so much in a detailed study of its separate elements as in the *regularities*, *mechanism*, *and result of their interaction* subject to the concrete conditions.

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* E. E. Leizerovich, "On Projecting Territorial Industrial Complexes in Regional Planning". In *Raionnaya planirovka i problemy rasseleniya* (Regional Planning and Problems of Settlement), Kiev, 1974, pp. 27-28.

** N. N. Kolosovsky, "Industrial Territorial Combinations (Complexes) in Soviet Economic Geography". In *Geografiya khozyaistva SSSR 1917-1947* (The Geography of the Soviet Economy 1917-1947), Geografgiz, Moscow, 1947, p. 138.

*** N. N. Kolosovsky, *Theory of Economic Zoning*, p. 142.

In accordance with the definition adopted in our work we consider the composition of a complex and the content of the problem of optimising its organisation to be the following: determination of the best variant (from the angle of the country's economy) of the exploitation of all types of resources, of the periods, rates, scale, and proportions of development, structure, and spatial organisation of all elements of the complex's economy and of its external and internal connections, subject to maximum economy of the input of social labour to meet the output targets of the specialisation industries and to provide the planned living conditions for the population.

The choice of approach for solving this problem and developing the modelling apparatus is mainly affected by the following features of a territorial industrial complex as a system:

(1) the complexity of its structure;
(2) the integrated nature (interconditionality) of its organisation and functioning;
(3) the open character of its functioning;
(4) its dynamism;
(5) its indeterminacy and stochastic character.

The structure of a territorial industrial complex. The complexity of the structure of a complex as a subject for modelling is expressed not only, and not so much, by the multiplicity of its constituent elements as by the variety of their interconnections and (which is particularly important) by the difference in their role and the regularities underlying its organisation and functioning. All the elements of a complex as a subject of modelling can be united in three large groups, viz. industry, infrastructure, and local resources, each of which in turn can be subdivided into separate components (see Diagram 1.1). Such a break-down is not universal but corresponds to the view of a complex taken here as a certain form of territorial organisation of production (mandatory involvement in the territorial division of labour, interconnected development of all the elements; and the existence of limits on the use of resources in short supply), and to the means adopted to represent the role and functional conditions of the separate elements of a complex in models.

Special attention was paid, when forming the groups and their sub-divisions, to the analysis of two attributes—(a) the rôle and connections of each of the elements being investigated in the set-up of the complex, and (b) their mobility and interchangeability.

Production. The basis of any complex is the sphere of material production, which comprises a group of specialisation industries, a group of complementary industries, and certain elements of the infrastructure. The organisational basis of the production sphere in a complex has become broad development of the division of labour within it, i.e. the specialisation of individual undertakings and concentration, combination, and co-operation of production.

The *specialisation industries* of a complex determine its place in the territorial division of labour of the country and of the economic region where it is located. These industries, and the period and rate of development, are determined by the make-up of the complex's resources, the national demand for them, the geographical conditions of the area, and the inputs of producing the product and delivering it to consumers. The basis of the specialisation industries is formed by one or more full or curtailed production cycles.*

As a rule, the kernel of the cycle is formed by the one or two leading lines of production that form the specialisation industries of the territorial industrial combination being investigated. The composition of the production cycle and its spatial organisation may alter, depending on the composition, economic importance, and character of the allocation of resources, the local production conditions, and the specific

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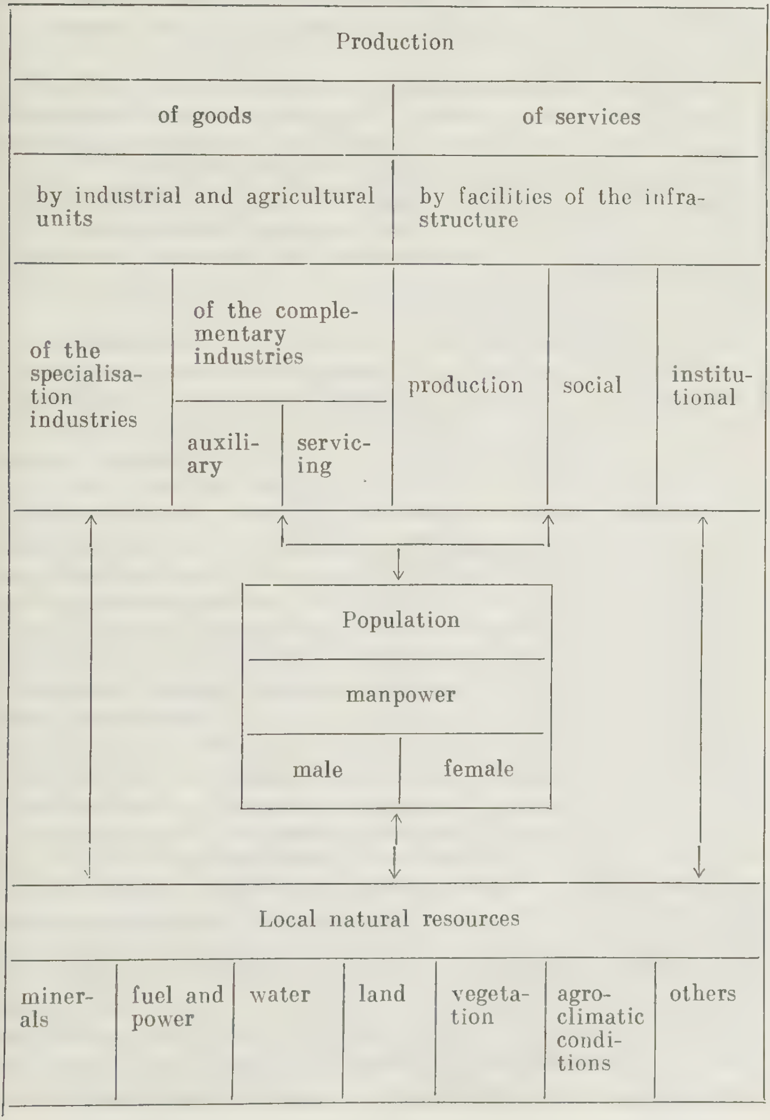
* The determining role in the shaping of a production cycle, in contrast to a normal technological cycle, is played by the economic relations between the enterprises of various industries as well as by their technological linkages.

Diagram 1.1. **The Main Elements of a Territorial Industrial Complex**



character of the separate production processes (even with an identical leading sector). The production cycles of complexes, in turn, are components of the bigger power and production cycles that form the basis of the economies of economic regions.

The specialisation industries of complexes are subdivided according to their economic importance and territorial characteristics into inter-regional and intraregional ones. Inter-regional industries usually include those whose output is intended mainly for shipment to other areas of the country, while the production of an intraregional industry is mainly consumed within its economic region. As a rule, the range of inter-regional specialisation industries is narrower than that of intraregional ones.

The industrial structure of a complex, i.e. the combination of its specialisation and complementary industries, is organised in accordance with its specialisation and natural and economic conditions. Several complexes with a similar specialisation will differ in industrial structure as a consequence of differences in the aggregate of local resources and conditions.

The economico-geographic position of a complex has a considerable effect on its structure (complexes of the northern latitudes, as a rule, have a narrower range of complementary industries than those located further south). The efficiency of a complex's industrial structure is thus determined not by the range of industries but by the economically justified combination of all the elements of both the production and the consumption spheres that ensures the most economic input of social labour to carry out the output programme of the specialisation industries and provide the stipulated living conditions for the population.

The second component of the production sphere of a complex consists of the *complementary industries*, which may be divided into two groups, auxiliary and service industries. The division is arbitrary, and it is sometimes difficult to draw a hard and fast line between them, but one must do so because the development of the separate groups of complementary industries is governed by different regularities and must be reflected differently in the models. In the course of the development of a complex's productive forces some of its complementary industries may become specialisation industries.

The auxiliary industries include those needed to provide the operating conditions of enterprises of the complex's specialisation industries. The structure, period, and scale of development, connections and location of auxiliary industries are determined by the needs of the specialisation industries, the size of the local resources, and the efficiency of building these industries within the complex compared with the cost of bringing similar products in from outside.

The service industries include those intended to provide the local population with food, goods of cultural, domestic, and everyday use, and prime necessaries. The make-up, scale of development, and location of these industries are determined as a rule by the complex's demand for their output, the specific nature of the raw materials, industry, or finished product, and by the indicators of the inputs of their production compared with the bringing in of transportable goods from outside the complex.

Thus, the range of matters dealt with for each of the elements in a complex's industry is different, subject to the specific conditions of their operation and the corresponding limitations. For specialisation industries the location of new undertakings and the scale of development of existing ones, and their connections, are fixed, subject to obligatory meeting of their output targets, while the composition, location, volume of production, and connections of the complementary industries are established subject to observation of the limitations for each source of local resources. The inputs for all production activities are clarified and made precise.

Infrastructure. We agree with the writers who include in the infrastructure those branches that provide services for all the other elements of the economy of a territorial industrial complex and for its population, but do not directly produce output in a physical, material form (exceptions being public catering, bespoke tailoring and dressmaking, and certain forms of public services and amenities). The level of development of the infrastructure has a substantial effect on the efficiency of the operation of units in the sphere of material production, and on the provision of labour, and largely characterises the maturity of a complex. The level of development and character of the location of the separate sectors and facilities of the infrastructure are also important in the organisation of a complex.

The infrastructure forms part of a complex as a major consumer of many types of output of the units of the sphere of material production, and of resources, especially multipurpose resources like capital investment, labour, water, land, etc., and not simply as a supplier of services. The existence of such close interconnections with the other elements of a complex dictates the need for a combined survey of the development problems both of the infrastructure and of the other elements of the complex, while its specific features single out the infrastructure as an autonomous object of study.

The representation of the infrastructure in models is mainly affected by the following features of its separate sectors and facilities: (i) its function in the complex and the character of its links with the other elements of the economy and with the population; (ii) its role in the territorial division of labour; and (iii) the character of its location and distribution.

We divide all the sectors of the infrastructure into three big groups: (a) production (railways, power transmission lines, pipelines, water mains and other municipal engineering works, and supply bases and warehouses); (b) social (passenger transport, trade, public catering, child-care and other children's institutions, general schools, housing, medical and communal facilities, and other service establishments); and, finally, (c) institutional (party and administrative bodies and agencies, scientific institutions, art facilities, specialised higher schools). In contrast to the commonly adopted classification, we also include in the industrial infrastructure the part of the complementary industries whose output is consumed by all the elements of the complex's economy and by the population (e.g. the building industry and heat and power stations).*

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* More recently many authors have rightly posed the need to take more account of the scientific and technical readiness of an area for development, which (it is suggested) should be considered as a special component of the infrastructure.

This grouping reflects the functions of the different sectors of the infrastructure, the character of the services provided by them, and their links with the other elements of the complex. Thus, the make-up, scale of development, and location of the production infrastructure are determined in the main by the demand of the units of the material production sphere, while those of the establishments of the social and institutional infrastructure are determined by the size and distribution of the population, and in part by the complex's place in the system of administrative territorial units. In addition, there is feedback, i.e. the effect of the level of development reached, of the existing location and the building conditions of new units of the infrastructure on the organisation of the spatial and industrial structure of the sphere of material production and of the system of settlement.

A distinguishing feature of the infrastructure is the non-transportability of its output, i.e. its orientation on supplying the market of strictly limited territorial units. This is particularly so for the sectors of the social infrastructure. But as a rule, there are elements of the infrastructure on the territory of any complex whose significance goes beyond the confines of one complex, e.g. sections of the trunk transport lines (railways, power transmission lines), higher educational establishments, certain administrative institutions, and so on. Their formation, unlike that of the other facilities of the infrastructure, is affected by conditions external to the complex. The level of development, shape, structure, and volume of the traffic of main-line (trunk) transport are determined not so much by the needs of the complex concerned as by national economic needs due to the territorial division of labour within the country, taking international exchange into account. The existence of infrastructure facilities of intercomplex and inter-regional significance within the limits of a certain complex makes it necessary to take account of the possibility of a significant increase in demand for the complex's resources as well as of exploiting certain reserves of these facilities (the carrying capacity of trunk lines, and so on).

A point character of location is typical of almost all the facilities of the infrastructure; they are built in places of concentrated demand within individual inhabited points. The volume and composition of the requisite services are determined by that, and also the demand for local resources and sources of supply to cover it. Railways, transmission lines, and other facilities of the infrastructure with a built-in linear character call for different treatment. The various combinations of the features just enumerated, and the force with which they are manifested in the different groups of the sectors of the infrastructure call for a differentiated approach and specific means of reflecting the conditions of their organisation in the composition of a complex. In this book only the most typical features of groups have been taken into account when dealing with practical problems, and sometimes of separate sectors of the infrastructure.

The demand for services of all sectors is determined by the solutions obtained while the distribution of services and the scale of development, location and connections of the facilities are made precise by the indicators of inputs. For the individual facilities of the infrastructure whose significance extends beyond the complex under study, a target for meeting the demand of territorial units of higher rank is introduced. The infrastructure of a complex is thus organised in accordance with the general level of development of its productive forces, while the location of its facilities is determined by the specific conditions of their operation and the demand of all the other elements of the complex's economy and of the population for them.

Population is the most important constituent of the productive forces. When a complex is being modelled the population is considered on the one hand as the producer of material wealth, and on the other hand as a major consumer of resources, of the output of the sectors of material production and of the services of the infrastructure. In that connection the complex's needs as regards the numbers and composition of the labour force, manpower reserves, and the conditions and ways of providing labour for all the elements of its economy are clarified first.

The outlays entailed in attracting and retaining labour are determined, taking into account the stipulated living conditions and the anticipated migration processes in the different parts of the complex (special attention being paid to analysis of the sectoral composition of the production units, scale of settlements, working conditions, and maintenance of the people's health from the angle of their effect on migration processes and build-up of population). The influence of the existing system of settlement on the location of new production facilities and of other elements of the economy are elucidated, and conversely the effect of the location of new production facilities on the shaping of the future system of settlement.

The numbers, composition, and territorial distribution of the able-bodied population not employed in social production are taken into account. From the results obtained the demand for labour is clarified, and its distribution among the sectors of the economy exercised, the basis of a scheme of settlement and peopling of the main inhabited points is outlined, and outlays on recruiting and settling labour are ascertained.

Local natural resources in the main condition the specific character of the process of the forming and production and spatial structure of each complex. But they only characterise the potential of the given area and the prerequisites for building a complex. The time taken to build each complex, the rate of building, and the scale of development and specialisation of the complex are determined by the national demand for its resources, to what extent its resources are in short supply, and the efficiency of exploiting them, allowing for both the economic, geographical, social, and strategic factors. The composition, scale, and readiness of the resources for exploitation thus, as a rule, determine the production structure of the complex, while their territorial concentration its spatial structure.

Each type of resource is analysed from the aspect of its possible involvement in the forming of the elements of the complex's economy. To this end the inputs connected (a) with definite modes of developing individual sources of supply, and of making natural resources ready for use, and (b) with their conservation and reproduction, are elucidated during investigation of the ways of exploiting them.

The natural resources group is divided into the following sub-groups: minerals, fuel and power, water, vegetation, and land. That division makes it possible to represent their links with consumers (industry, agriculture, the infrastructure, and the population), their transportability, renewability, and other specific conditions of their exploitation, and problems of the conservation and reproduction of individual types of resource, and sometimes also of their sources. The sources of resources are selected by natural resource, and the scale and variant for working them and the scheme of links with consumers determined, and inputs estimated from the results of the solution.

Climatic, agroclimatic, and recreation resources, the state of the atmosphere, and certain other natural conditions are distinguished as an autonomous group, and are represented in the forming of the initial information and data, in the definition of limitations, input coefficients, yield indices, and outlays on attracting and retaining population.

Matters of the protection and reproduction of resources have a special place among the problems of a complex. Their sources are treated not only as suppliers of raw materials, fuel, and production and living conditions, but also as major consumers of material resources, above all of capital investments. Many of the conditions (limitations) of the problems are formulated with allowance for the requirements of nature protection. As a result, variants are obtained for the carrying out of certain measures for the conservation or restoration of resources. In that connection one of the most pressing problems of science is to develop a theory and method for quantitative evaluation of all types of resource.

The economico-geographic position of a complex, which has a substantial effect on the organisation of its economy in both time and space, can be arbitrarily included among its resources. But many difficulties arise when one tries to take its effect into account when modelling a complex. The theoretical problems of economico-geographic position have been deeply studied by Baransky and developed in the works of Mayergoiz, Pokshishevsky, Konstantinov, Saushkin, and other Soviet geographers, the first, interesting studies on quantifying it have been published.

The difficulty of introducing this factor into the model of a complex is that its assessment cannot be quantified in advance (except by using very crude coefficients) and remain unaltered irrespective of how, when, and with what facilities will not only the given area itself but also adjoining parts of the region be developed.

In that connection a quantitative assessment of the role and significance of the economico-geographic position can be obtained in the course of the solution of the problems, and for certain concrete conditions. It will be the more exact the more accurately the main characteristics describing the specific features of the area concerned are represented, and the fuller their effect on the organisation and functioning not only of the complex's economy in general but also of each of its elements (and sometimes of their components), and of the population, are taken into account. The indicator of the assessment of economico-geographic position, in our view, is not initial information for the problem of forming a complex, but one of the results of solving them. The assessment is obtained in accordance with the solution obtained, and its value is correct only for a definite variant of the development of the area, and for a definite time period.

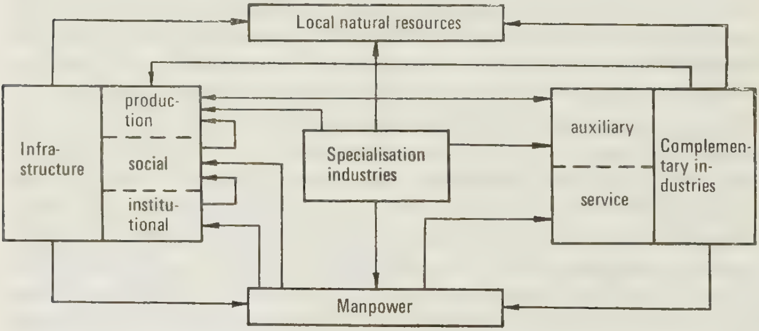
In this work we were only able to reflect the effect of the economico-geographic position on the organisation of the industrial and spatial structure of a complex through a system of indices reflecting the infrastructure facilities of the area, its industrial-transport links, and the conditions for attracting and retaining population. The result of the solution obtained showed the possibility of quantifying the specific nature of the area's economico-geographic position, of bringing out the role of the separate factors considered in the forming of this assessment, and of determining the effect of the position on the structure of the economy obtained as a whole, and on the location of its separate elements.

The multicomponent make-up of a complex, the great diversity of the conditions for the organisation and functioning of each component, and the difference in the content of the problems that have to be decided for the separate elements, make it necessary to divide the general problems of optimising the organisation of a complex into parts (or stages), and to develop groups of models.

The integrated character (mutual conditionality) of the organisation and functioning of the elements of an economy. This is an important feature of a territorial industrial complex. The complementary industries and the infrastructure have an exceptional role among all the elements of an economy. The dependences within a complex are manifold, and its maturity is characterised by the level of development of these connections, which are realised along the lines of the production process, the supply of materials and equipment, and joint use of resources and of the facilities of the infrastructure.

Connections arise within any complex between the facilities of the specialisation industries and the group of complementary enterprises, between industry and the infrastructure, between and within the facilities of the separate types of infrastructure, and between local resources and manpower and all the elements of the economy (see Diagram 1.2).

Diagram 1.2. **The Interconnections Within a Territorial Industriral Complex That Determine Its Organisation (Scale of Development; Intensity of Functioning or Exploitation)**



The composition and scale of development of the specialisation industries are determined by the delivery targets for output, and in turn have a decisive effect on the composition, rate and scale of development of auxiliary enterprises and the units of the production infrastructure, and largely determine their location. The development of the production infrastructure in turn calls for supplementary development of the auxiliary industries. The specialisation and auxiliary industries and the production infrastructure need labour and predetermine the basic system of settlement. The population is formed in accordance with the demand for labour (allowing for the specific features of the composition of the industries and the conditions of the area). The size and location of the population determine the composition, scale of development, and location of the service industries and of the social and institutional infrastructure. The service industries, and the social and institutional infrastructure, evoke an additional demand for the services of the production infrastructure and for the output of the auxiliary industries. After the general scale of development of the infrastructure has been discovered, allowing for the targets for facilities of intercomplex significance and complementary industries, the total demand for labour and local resources is found.

The connections enumerated determine the inner mechanism of the formation of a complex as a whole and are taken into account in the model by representing the conditions of the organisation and functioning of each of its elements. Thus, when the conditions of the formation of auxiliary industries are being represented, their links with the specialisation industries and with the other facilities of the auxiliary industries, service industries, and production infrastructure are allowed for. When planning the population and determining the scale of the exploitation of local resources, their links with the specialisation and complementary industries and with the infrastructure as a whole are taken into account.

The dependences within a complex are very important for understanding the process of its organisation, but are also very difficult to represent in the model, which is due not only to their density but also to the great variety of their content, forms of manifestation, and means of realisation. It is simpler to represent the connections due to exchange of products, more complicated to represent joint demand for services of the infrastructure, and very difficult to represent the joint exploitation of multipurpose resources. It is particularly complicated to represent the indirect connections (through intermediate links), and the regularities giving rise to new or recurring connections (e.g. the composition and scale of development of the facilities of the social infrastructure depend on the size of the population, but in turn call for labour, with the result that the population increases in size, and so on). The need to take the intracomplex links into account is the reason for the marked increase in the scope of the problems and the complication of the models' structure. In the solution obtained the following are discovered for these links: their composition, scope, direction, and means of realisation (excluding intraproduction links); and the separate indicators are made more precise.

Intracomplex connections largely determine the efficiency of one variant or other of a complex's formation. But representation of the effect of integration in the models, i.e. of the effect of joint location and intracomplex connections, is still the most difficult and least developed problem of the modelling of a complex. An effect of territorial concentration arises in connection with the total reduction of the expenditure of resources and inputs for the building of both enlarged production facilities and enlarged infrastructure facilities. Non-linearity of the changes in inputs in relation to the volume of production or exploitation of resources is characteristic of the indicators of the effect of territorial concentration.

The open character of the functioning of a territorial industrial complex. The existence of internal and external stable technological, production, economic, demographic, information, and organisational connections is a *sine qua non* of the normal functioning of a territorial industrial complex as a complex multifunctional formation and component of the country's economy. The basis of the rise of connections is the difference between the prerequisites and conditions of development of the productive forces of individual areas, the territorial division of labour, and improvement of the forms of organising production and developing an area.

As a complex's productive forces develop the composition, trend, and effect of connections on its formative process alter. It may happen that, given target-oriented planned development of a complex, there will be a redistribution of the centripetal and centrifugal operative forces of its formation process, a strengthening of its inner connections, and the forming of a complex aggregate of closely interconnected elements.

Centrifugal forces, namely. the state's external influence or demand for certain types of output, give rise to new facilities in the specialisation industries, which are included into the all-Union production system, so that the complex proves more and more to be linked with the general system of territorial division of labour. Connections are gradually extended in scope and space, which largely determines the rates, trends, and scale of development of the complex's productive forces.

Parallel with intensification of external connections the specialisation industries' demand for local resources grows, which gives rise to an increase in population and development of complementary industries and of the infrastructure. If this happens according to a plan previously worked out from the standpoint of the national economy, it will lead to a strengthening of internal connections between the elements of the complex, with a clearly expressed centripetal trend in their operation. A complex's connections are thus a reflection of its two most important features: namely, involvement in the territorial division of labour, and unity of all its elements.

The external connections of a complex determine its place in the country's economy, and its specialisation in the inter-regional and intraregional division of labour. As a rule, they are bilateral: namely, from the national economy to the complex (direct), and from the complex to the economy (feedback). The direct connections reflect the country's demand for the complex's resources or end products (the production target for the specialisation industries) and the composition and quantity of resources (products) that the national economy can provide the given complex. The feedback connections contain information on the complex's resources and possibilities, on the inputs by which it can fulfil the given programme, and the list of resources (products) of other complexes (regions) of the country that it needs for that purpose.

Complicated external connections arise between complexes within a single economic region. They are manifested not only in reciprocal exchanges of output, joint exploitation of natural resources and communications, but also in the existence of competition between them as elements of a single economic organism. Since several complexes of a region may be involved in deciding certain matters facing the region, competition can develop among them over the location of similar industries, the allocation of production programmes or of resources in short supply, and of output (similar or interchangeable).

There are also other fields of interconnections between the territorial industrial complexes of a region. There is, for example, a definite, objectively dictated sequence in the organising of individual complexes, in which the more mature ones act as supports for newer ones. Performance of this support role necessitates the development of certain industries (building, generation of electricity, food, repair facilities, and so on), taking the needs of the new complexes into account. Or there may be another relationship. Exhaustion of the resources of an already developed complex, or a shortage of them, may necessitate the building of separate lines of production in the new complexes.

The existence of connections between each complex and the others in its own or neighbouring economic regions excludes any possibility of optimising it outside the economy of the country, and in particular, outside its own economic region. This conditioned the development of the first stage of our proposed approach to optimising the organisation of a territorial industrial complex: namely, investigation of the whole system of complexes of a given region.

In order to bring out the role of external connections in shaping the industrial and spatial structure of a complex, in contrast to its intracomplex connections, only production-transport connections are taken into account, i.e. links dictated in the main by the production process and realised by one of the forms of freight transport or by transmission lines. The content, mass scale, and stability of the flows, and the means of effecting them, are allowed for in the choice. Attention is paid to the stable numerous physical connections effected by railways, water transport, pipelines, and transmission lines, i.e. those connections whose volume, direction, and structure affect the location of facilities, the development of the infrastructure, and the expenditure of resources.

External connections, unlike internal ones, are fixed in advance, as a rule. At the level of a complex, therefore, their role comes out in the organising of the complex, and the separate indicators and means of realising them are elaborated within the limits of the complex.

The dynamic character of the functioning of a complex. The start on a territorial industrial complex is preceded by major research and development work. The process of building the complex usually takes several decades, within which we can distinguish periods of preparation of the infrastructure, and of intensive building of the main facilities, and finally a period of operation and functioning. All the periods are interconnected and the transition from one to the other may take a long time. In addition, the period from building the infrastructure to intensive construction does not exclude the functioning of some of the facilities already built; and as a rule, the building of some industries may be completed, others may be reconstructed, and a start made on building new ones in the period of functioning of the whole set. The process of formation of a complex and perfecting its structure as a territorial industrial system is thus, in practice, continuous.

In spite of all the arbitrariness of this periodisation, we need it for modelling the formation process because, without it, it is impossible to reflect the different problems facing the system in the different stages of its development.

The dynamic aspect of analysis of the formation of a complex assumes that the following is ensured:

(1) proportional organisation not only of the production and spatial structure of the complex as a whole but also of its separate interconnected elements, allowing for their location;

(2) an allocation of resources efficient from the standpoint of the complex as a whole and for the whole calculated period;

(3) utilisation of the capacities built and reallocation of resources in time and space;

(4) taking account of the possible stage-by-stage character of the organising of the complex in time and space;

(5) adaptation of the complex to the situations that can arise through adopting solutions for the development of areas gravitating towards the complex as well as for the complex itself;

(6) the possibility of expanding facilities, and the exclusion of cases in which the location of some facilities proves

a reason for limiting the capacities of others or necessitates transfer of capacities already built.

The main difficulty in modelling the process of the forming of a complex as a dynamic system comes from the fact that, in addition to the general problems of allowing for the dynamics of the economic problems (effect of the post-planning period*, lag, the preparation of information, etc.), specific difficulties also arise. Above all the complex's separate elements and their components differ in the way their development proceeds as well as in degree or level of dynamism. The same factors, which determine the dynamic character of the system (technical progress, the stage character of the solution of the economic problems), affect the process of the building and functioning of the complex's separate elements and their parts differently.

In working out ways of reflecting the dynamics in the modelling of a complex, we have taken the following into account:

(1) the character of the problems: namely, determination of the siting variant, effective from the angle of the whole planning period and allowing for the time taken to build the various facilities (determination of spatial structure and settlement); determination of the variant for building separate facilities by stages of the planned period, taking the end effect into account (i.e. organisation of the building industry, transport system, etc.);

(2) the character of the development process, i.e. whether smooth and even (growth of settlements, exploitation of local mineral and vegetation resources), or intermittent and irregular (reconstruction of railways, exploitation of water resources);

(3) the character of the change in indicators of specific (unit) inputs during the building of the complex, i.e. whether increasing (exploitation of local resources, environmen-

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* This point is of great significance for modelling the organisation of a complex because it is necessary to find a way of determining and reflecting in the models the effect that will be brought about in the future not only in the complex itself but also in the areas gravitating towards it. It is particularly difficult to estimate the investments in the infrastructure and on protection of the environment since some forms of the return from them will reduce outlays on developing the productive forces of other complexes even beyond the planning period.

tal protection) or decreasing (building of elements of the infrastructure);

(4) the character of the adoption of decisions allowing for the uncertainty of the conditions, i.e. whether final for the whole period (e.g. location of facilities, choice of resources, etc.) or for the stages of the building of the complex (e.g. the load and connections of the building industry, the building of facilities of the infrastructure).

Two main types of problems connected with allowing for the dynamic properties of a complex as a territorial industrial system can be distinguished. By means of the first type we determine *what to locate and where*, and by means of the other type *when to build the complex's facilities and how*. In the first case the strategy of developing the area is worked out, mainly taking the factors of location into account; the decisions of principle on these matters (the location of the facilities) must be taken immediately for the whole period, especially those for the objects of the first stage. In the second case the strategy of implementing the programme in time is determined, taking into account, in the main, factors of organisation and production techniques. These matters are more dynamic, and their solutions can be decided by stages and periodically corrected. The production and spatial structure of the complex is determined by resolving the first type of problem, i.e. matters that require taking the final effect into account, and also the effect from the angle of the whole planned period being assessed, while solution of the second type of problem fixes the time, rates, and means for building the separate production units and exploiting resources, i.e. answers questions that must be resolved not only at the end of the whole period but also at its various separate stages.

Production is not only the basis of a complex but also the most dynamic part of it. Spatial structure is less mobile and is largely built up through the effect of the sectoral structure and shaped, as a rule, after completion of the work on building the main production facilities.

Some of the dynamic features of a complex can also be taken into account when static models are employed, by correcting the constraints, input indices and making a series of calculations. But the possibilities of static models are limited, and dynamic ones are also needed so as to solve the whole set of problems of the organisation of a complex. The specific feature of our problems is that it is necessary in many cases to analyse processes simultaneously that may alter in both time and space. A dynamic spatial model, however, is not needed in all cases. A substantial part of the problems can be solved by means of static models, dynamic point and grid models. In that case dynamic spatial models are used, in our view, to reinforce analysis of the formation process of the complex as a whole.

The dynamism of complexes is also expressed in the fact that their production and spatial structure alter and that, as we said above, both their area and the contours of their boundaries may be altered. The alterations are brought about by both endogenous and exogenous factors, i.e. by development of the productive forces of the complex itself and by the formation of other complexes in the given region and in the country.

As a result of this interaction there is a bilateral effect: namely, of the individual complex on the system of territorial industrial complexes, and of the system on the individual complex. In that connection yet another supplementary requirement is posed for dynamic models, that of taking the dynamics of the external links into account. They must reflect the conditions of the process of adapting the complex's economy to changes in the system of complexes, and those of the system's adaptation to the changes that may occur in separate complexes—all of which once more emphasises the need to analyse a complex as an open territorial industrial system as well as a dynamic one.

Uncertainty and stochastic character. A territorial industrial complex, like any integrated economic system, has inherent features of uncertainty and probability. This is explained by the difference in the nature of the factors determining the specific character of the organising of each complex, the complexity of its economic structure, the difference of the ways of formation of individual units, the open character of its functioning, and the duration of the development process. These features are intensified when modelling complexes in connection with the defects of the mathematical apparatus developed and the inadequacy of the initial information. Scientific and technical progress, and the periodic development of new economic problems, also greatly alter the conditions of the formation of individual complexes.

The advances of science and technical progress make it possible to draw new types and sources of various resources into economic circulation and to improve the efficiency of their exploitation. As a result, the potentialities of individual complexes undergo reassessment and their place in the territorial division of labour is reviewed, and that leads to a change in the set of the facilities, production structure, external and internal connections, and the time, rates, and plans for the formation of individual complexes.

As a consequence of the objective character of territorial industrial complexes as specific forms of the organisation of the productive forces, the process of forming them has acquired "a dialectical, dualistic character, i.e. determinacy of the main, relatively long-term trends of development through the effect of objective trends, and deviations from this development through the combined effect of a large number of casually indeterminate relationships".* Those conditions for the organisation of a complex are the more definite which are due to operation of the objective laws of development of nature, the productive forces, and the relations of production. But because our knowledge of the manifestations of these objective laws in concrete conditions is deficient, we can reflect the effect of some factors quite accurately in models, of others less accurately, and of still others only conditionally or arbitrarily. There are also factors (e.g. certain complicated geographical phenomena) that can be taken into account only very roughly, as many geographers have rightly remarked. All that is reflected in the varying reliability of the data used to formulate and solve practical problems. The better the area and production of future complexes have been studied, the greater is the certainty of the initial information.

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* L. A. Melentiev, "Initial Premises of the Problem of the Indeterminacy of Optimum Solutions in Large Power Systems". In *Faktor neopredelennosti pri prinyatii optimalnykh reshenii v bolshikh sistemakh energetiki* (The Uncertainty Factor in Adopting the Optimum Decisions in the Major Energy Systems), Vol. 1, Irkutsk, 1974, p. 13.

The existence of a different degree of certainty of the conditions for developing a complex, and the use of determined, stochastic, and incomplete initial data, do not permit us to get unambiguously definite answers about the functioning and development of big systems, give rise to zones of uncertainty of the optimal solution, and call for rethinking of the results obtained and of the essence of their mathematical economic analysis. It is important for forecasting pre-planning research not so much to analyse the optimum plan (which holds only for definite conditions) as to analyse the system's behaviour given changes of conditions and adaptation of the complex to different situations.

The stochastic, uncertain character of a territorial industrial complex is thus one of its very important characteristic features as an object of modelling, one that has to be taken into account at all stages of the investigation, i.e. when formulating the problems and preparing the information, and especially when analysing the results obtained.

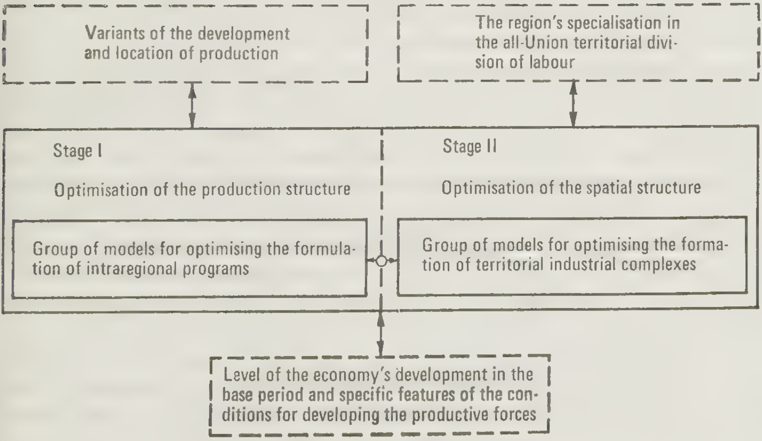
3. THE APPROACH TO MODELLING TERRITORIAL INDUSTRIAL COMPLEXES

One of the three main parts of the system of mathematical economic models of long-term territorial industrial planning proposed by our Institute is the sub-system of regional models intended for detailed study of the paths of most effective development and allocation of the productive forces of regions (economic regions and their segments). These models are needed so as to analyse the regional features of the development of separate territorial units, ways of improving the efficiency of the exploitation of resources, and the economic connections and spatial organisation of the economy.

We suggest that the regional problems should be tackled after the national economic and Union sectoral ones, when the region's place in the solution of major national problems and the following points are known: its specialisation in the all-Union territorial division of labour) pattern of the specialisation industries, delivery targets for their output, and the distribution of shipments by destinations); the composition and assignments for the development of infrastructure elements of inter-regional importance; the resources of inter-regional importance that are in short supply; assignments for the region's involvement in measures of an inter-regional scale to protect and conserve the natural environment and to provide a certain standard of living for the population; inputs to produce the output of the specialisation industries by which the region's place in the territorial division of labour is determined. In addition, we also then know the possible variants and techno-economic indices for the development and location of the facilities of the specialisation industries, agricultural production, and elements of the infrastructure, the conditions for developing the area and exploiting local natural resources, and the outlays on attracting and settling population.

The pre-planning research into possible lines of development of an economic region's productive forces and economic structure, employing econometric models, must be broken down into two successive stages: namely, (a) initial analysis of the productive structure of its economy and sub-divisions, and (b) analysis of its spatial structure (see Diagram 1.3).

Diagram 1.3. **Optimisation of the Production and Spatial Structure of an Economic Region's Economy**


Variants of the development and location of production
The region's specialisation in the all-Union territorial division of labour
Stage I
Optimisation of the production structure
Group of models for optimising the formulation of intraregional programs
Stage II
Optimisation of the spatial structure
Group of models for optimising the formation of territorial industrial complexes
Level of the economy's development in the base period and specific features of the conditions for developing the productive forces

The task of the *first* stage is to bring out the main intra-regional programmes (problems), define the composition and production structure of the economy, optimise the proportions of the development of its components, the allocation and exploitation of multipurpose resources and the development of the production and social infrastructure. The results obtained bring out data that will be taken to clarify the specialisation of economic regions by means of an interregional intersectoral optimisation model.

Since the basis of the spatial structure of a region is formed by the aggregate of territorial industrial combinations, defining it means to determine the spatial structure of the whole system of intraregional complexes. But it is impossible to do that without resolving matters of principle of their creation and functioning, i.e. without optimising the organisation of each of them. For that purpose the problem of optimising the formation of the combinations is treated as a component of the pre-planning research, while the combinations themselves are taken as elements of the country's territorial system.

The task of the *second* stage is thus primarily one of delineating the main intraregional territorial industrial combinations, determining their place in the inter-regional and intraregional division of labour, optimising the direction and scale of the exploitation of resources and of the structure and location, connections and spatial organisation of all the elements of the economy. At this stage many of the indices that have been used in solving the problems of the first stage are also made more precise. In that connection, optimisation of the structure of the region's economy as a whole can be considered completed only after the results of both stages have been mutually tied up and co-ordinated with the data obtained from solving higher problems using an inter-regional intersectoral optimisation model. A group of models for optimising the organisation of territorial industrial complexes is proposed as the instrument for investigating the second stage (see Diagram 1.5).

In Soviet planning practice the national economy is looked at from three aspects: namely, global, sectoral, and territorial, i.e. the whole economy (system) and its parts (sub-systems of its sectoral and territorial elements) are studied. Each of the sub-systems distinguished consists in turn of individual links.

The territorial structure of the national economy can be represented in the form of mutually interconnected territorial industrial combinations of various rank. The division is based on analysis of a combination of two attributes: namely, production and territory. Thus, territorial industrial combinations are distinguished as autonomous units for which it is possible, when drafting the pre-planning documents of a particular level (depending on the rank of the combination) and the corresponding system of management, to obtain the maximum effect through integrated development and rational location and organisation of the economy. The rank of each of the combinations distinguished is determined by its place in the solution of the national problems and in territorial industrial planning, by the level of development of the productive forces and structure of production, and by the composition and intensity of its external and internal economic ties. The specific features of the corresponding forms of territorial organisation of production and units of economic zoning and national economic planning are inherent in the combinations of each rank.

One of the ranks of territorial industrial combinations is the economic region. Its spatial structure, in turn, is formed by an aggregate of combinations of two ranks: namely, large territorial industrial complexes and industrial centres. The complexes determine the face of the region. To delineate a complex and determine its structure means to determine the optimum variant of the scheme for the territorial structure of the economic region's economy as a whole. Several complexes form the basis (skeleton) of a region, and each complex in turn unites the territorial industrial combinations of industrial centres. The main features of a complex as a specific form of the territorial organisation of production, i.e. as an organic unity of industry, area and its resources, and infrastructure, show up more clearly in territorial industrial combinations of a given rank than in economic regions. Territorial industrial complexes are large territorial formations with big potentialities as regards the building of integrated multisectoral production combinations, for which the existence of many connections, both intrasectoral and intersectoral, is characteristic, and also of connections due to utilisation of the infrastructure of intercomplex significance.

Industrial centres coincide territorially with the boundaries of big towns, or embrace a system of interconnected inhabited points. The basis of the organisation of both individual centres and agglomerations of centres is the exploitation of certain types of local resources. The composition of the resources and the character of their exploitation, and the economico-geographic location of centres determine their place in the territorial division of labour. The specialisation and structure of the economies of individual centres and agglomerations are much narrower and simpler than those of a complex. With centres, unlike complexes, it is the intrasectoral connections that predominate as a rule, rather than the intersectoral ones; a community of area and connections plays a greater role as regards joint exploitation of local resources and elements of the social infrastructure.

A definite level of development of the productive forces is needed in order to form a territorial industrial combination of the rank of a complex. In Siberia, as in other regions of the USSR with large undeveloped reserves of territory, the boundaries of neighbouring complexes (unlike those of territorial administrative units) may not abut onto one another. There are reserves of land between them that still do not fall within any one of them. The complexes include only that part of the territory on which elements of the given complex are being organised. Only after the resources remaining outside a complex have been determined and the line for their exploitation and possible production relations have been determined can it be decided whether or not to include them in one of the adjoining complexes, which may lead not only to an increase in the complex's area but also to an extension of its specialisation. The scope of effective enlargement of a complex, however, is limited, and differs in individual parts of the country. An opposite process is also possible: namely, that of forming new complexes by breaking down existing ones. In general, the problem of the optimum size of complexes has not yet been studied, and calls for detailed investigation.

The area and boundaries of a complex are thus largely determined by the level of economic development of a region's territory, the composition and character of the location of local resources, and the specific character of the specialisation industries, all of which depends on the extent of its production and general economic connections, and the degree to which the periphery is attracted to the nucleus of the complex, and may be altered as the productive forces of both the complex itself and the surrounding area develop. The criterion for drawing the boundaries of a complex is the indicator of the significance of the connections between any point of the area studied and the industrial, transport, and administrative centres of neighbouring complexes.

It is impossible to optimise the development of a complex by means of a single model, since one is then trying to solve problems of a different level of planning at one and the same time, each of which problems necessitates taking account of a different set of factors of a different degree of differentiation. The separate problems of organising a complex are dissimilar in nature, character, and degree of interconnection. To solve them it is necessary to consider not only a different set of factors but also the application of different forms of taking them into account. In that way the logical scheme for studying complexes is mapped out, and also the approach and the instrument for the investigation, i.e. the mathematical economic model for optimising their production and spatial structure.

We took the following premises as the initial position when developing our approach to optimising the organisation of a territorial industrial complex.

1. There is centralised direction of the economy with considerable freedom for decision-taking by the local authorities, that is to say, the main parameters of the organisation of territorial units of any rank are determined at a higher level, while determination of the ways of shaping the area's economy as a whole is reserved to the appropriate local territorial administrative authorities.

2. The main pre-planning document for an economic region is the general plan for the development and allocation of the productive forces. One of the stages in drafting this general plan must be optimisation of the formation of territorial industrial complexes.

3. Optimisation of the development of a complex is understood as determining the best variant (from the standpoint

of the national economy) for employing natural resources, labour, and other resources, the best periods, rates, scales, and proportions of development, structure and spatial organisation of all elements of the complex's economy and of all its external and internal connections, given maximum economy of inputs of social labour to meet the output targets of the specialisation industries and provide the stipulated living conditions for the population, protect and conserve the environment and reproduce resources. That definition is, at the same time, the formulation of the general problems of optimising the development of the complex.

4. The organisation of a complex is optimised by finding the best combination of variants of the production structure, external and internal connections, and development and location of all elements of the economy, settlement, and exploitation of resources.

5. There is a system of models of long-term territorial industrial planning; and the pre-planning research into the structure of the economic region's economy is carried out by stages (see Diagram 1.3).

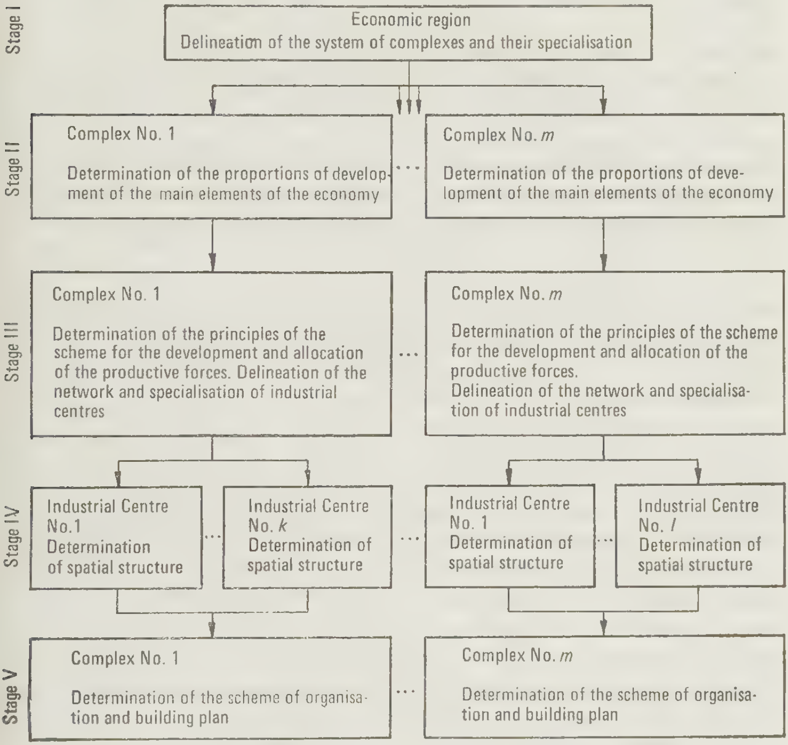
6. Because a complex is a structurally integrated, open, dynamic, stochastic economic system, the problems of optimising its organisation must be solved in stages, employing mathematical economic models of a particular type.

Underlying the suggested approach is the principle of the combination of enlarged and detailed analyses of the complex, gradual transition from the solution of general questions to partial ones, with their subsequent generalisation, i.e. from the problems of the organisation of an economic region's whole system of complexes to those of its separate complexes and their components (the separate elements of the economy and the territorial industrial combinations within the complex), and then the return once more to the complex as a whole.

Five consecutive stages are proposed for tackling the general problems of optimising the organisation of a complex (see Diagram 1.4).

The problems of all stages are solved subject to obligatory meeting of the production targets of the specialisation industries and development of elements of the infrastructure (the significance of which goes beyond the confines of the

Diagram 1.4. **Sequence and Aims of the Stages of Research**


Stage I
Economic region
Delineation of the system of complexes and their specialisation
Stage II
Complex No. 1
Determination of the proportions of development of the main elements of the economy
Complex No. *m*
Determination of the proportions of development of the main elements of the economy
Stage III
Complex No. 1
Determination of the principles of the scheme for the development and allocation of the productive forces. Delineation of the network and specialisation of industrial centres
Complex No. *m*
Determination of the principles of the scheme for the development and allocation of the productive forces.
Delineation of the network and specialisation of industrial centres
Stage IV
Industrial Centre No.1
Determination of spatial structure
Industrial Centre No. *k*
Determination of spatial structure
Industrial Centre No. 1
Determination of spatial structure
Industrial Centre No. *l*
Determination of spatial structure
Stage V
Complex No. 1
Determination of the scheme of organisation and building plan
Complex No. *m*
Determination of the scheme of organisation and building plan

area concerned), limitations on resources in short supply, and requirements allowing for the specific character of both the functional facilities and the separate parts of the area being studied. Such a posing of the conditions does not, however, exclude the possibility of the composition of the specialisation industries and the delivery targets for the separate types of output, and the scale and directions of the exploitation of resources, being made more precise. This is done when it appears necessary, from the results of the solutions, to alter the techno-economic indices previously used, and it proves (during the second solving of the higher-level problems with corrected data) that it is more expedient to site one industry or another in other complexes or industrial centres. As a result of the solution obtained, new ways of exploiting individual forms and sources of resources may be brought out.

Because each complex is a link in the country's economy, only that complex can be considered optimum within which the state output programme of the specialisation industries can be met, the requisite living conditions for the population created, nature protected and resources restored, with maximum economy of inputs of social labour. In that connection we adopted minimisation of the normalised aggregate inputs on the organisation and functioning of the economy of the complex being studied as the main criterion of optimality at all stages of our research. For that the effect of location was determined at regional level, of location and concentration at complex level, and of location, concentration, and organisation of the economy at industrial centre level. The adoption of this criterion as the basic one does not exclude the possibility of employing other criteria when dealing with certain matters connected with the organisation and functioning of complexes (the structure of the criterion for each problem is given in Chapters 4 to 10). Correspondence to the national economic optimum is achieved as a result of all the main parameters of the freedom of solutions, limitations, and conditions being derived from the solutions of the problems at a higher level of the system of optimal territorial industrial planning.

At stages of the solution three main types of objects of study are singled out, as follows:

(1) territorial taxonomic units: areals, sub-areals, and sectors;

(2) functional elements of the economy: production (specialisation industries, complementary industries, both auxiliary and service); infrastructure (production, social, and institutional); manpower and natural resources;

(3) production and transport connections (external and internal).

The classification adopted is due to the need to allow for the specific features of each object and its place in the organising of the area's economy, and to the nature of the problems of optimising their building and functioning.

Joint survey of all the elements of the economy, freedom in allocating resources, and allowance for the multilateral connections between the objects of research are a main feature of the approach proposed.

The possible set of territorial taxonomic units, and their characteristics, and the composition of and variants for exploiting local resources are determined before solution of the problem, at the stage when it is being formulated. During optimisation of the spatial structure of the economy the areals (sub-areals, sectors) for locating elements of the economy, and the scale and direction of the use of resources, are decided, and the indices of inputs made precise. Neither complexes nor industrial centres are given in advance, but are delineated in the solution from analysis of the connections of the economy's elements and the character of the exploitation of resources.

All the elements of a territorial industrial complex—industry, infrastructure, local natural resources, and population—are considered during solution of the problems of optimising its organisation. But the scope and itemisation of their analysis differ. As a rule, there are three levels of detailing the representation of the objects of research corresponding to the three main ranks of territorial industrial combination (region, complex, industrial centre). The detail of the research increases on the whole from region to industrial centre, the objects surveyed being broken down and the content of the analysis altered; rather less attention is paid to matters of the organisation of the region's specialisation industries, and more to the complementary industries, infrastructure, exploitation of resources, and nature conservation (see Table 1.1).

The sub-areal is taken as the main territorial taxonomic unit, i.e. a compact space suitable for industrial and civil building and agricultural exploitation, possessing certain conditions different from the surrounding territory for the building of functional facilities. At regional level, sub-areals close in their characteristics are united into areals, while at the level of an industrial centre sub-areals are divided into sectors. It is taken that an areal, or an aggregate of areals, is the territorial basis for forming a complex, a sub-areal for the organisation of an industrial centre,

Table 1.1.

Differentiation of the Objects of Research by

Territorial industrial combinations under review	Objects of Study		
	Territorial taxonomic units	Industries	
		specialisation	complementary
Economic region	Areal	aggregated facilities or large enterprises	auxiliary
Complex	Sub-areal	enterprises	auxiliary enterprises or lines of production
Industrial centre	Sector	enterprises and their parts	auxiliary and service enterprises and plants

and a sector for the organisation of an industrial zone, microdistrict (neighbourhood), recreation zone, or the administrative junction of an industrial centre. The functional facilities of the economy are itemised in a similar way.

In order to optimise the organisation of a complex a large quantity of diverse initial information is required. Its composition and differentiation in accordance with the rank of the combination and stage of the solution of the general problem are given in Chapter 2, and in the description of concrete models (Chapters 4-10).

Rank of Territorial Industrial Combinations

Infrastructure	Resources		Production-transport connections
	manpower	natural resources	
interareal sections of communications and industrial units of inter-regional and general regional significance	total numbers in the gravitation zone of the areal	sources (by types of resource)	flows of inter-regional and interareal freights
inter-sub-areal sections of communications, enterprises, and plants	numbers of men and women	sources (by types and categories of resource)	flows of inter-complex and inter-sub-areal freights
intracentre communications; branch lines and sidings; enterprises and complexes of establishments serving the public	numbers of men and women	sources (by types and categories of resource)	flows of passenger traffic within centres

4. THE TOOL FOR AND STAGES OF OPTIMISING THE ORGANISATION OF A TERRITORIAL INDUSTRIAL COMPLEX

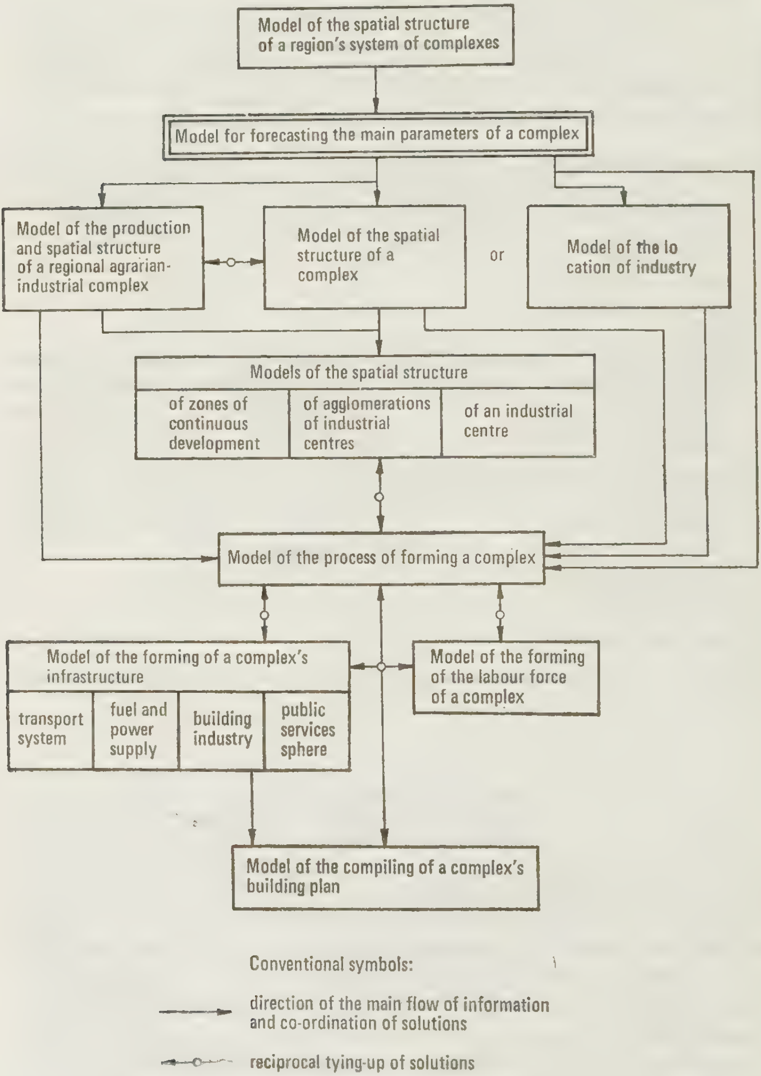
The tool of research. A group of models for optimising the organisation of a territorial industrial complex is proposed as the tool for solving the problems posed (see Diagram 1.5). The following are the main elements of the group:

a model of the spatial structure of a region's system of complexes;

a model for forecasting a complex's main parameters;

a model of the spatial structure of a complex;

Diagram 1.5. **Group of Models for Optimising the Organisation of a Territorial Industrial Complex**


Model of the spatial structure of a region's system of complexes
Model for forecasting the main parameters of a complex
Model of the production and spatial structure of a regional agrarian-industrial complex
Model of the spatial structure of a complex
or
Model of the location of industry
Models of the spatial structure
of zones of continuous development
of agglomerations of industrial centres
of an industrial centre
Model of the process of forming a complex
Model of the forming of a complex's infrastructure
transport system
fuel and power supply
building industry
public services sphere
Model of the forming of the labour force of a complex
Model of the compiling of a complex's building plan
Conventional symbols:
direction of the main flow of information and co-ordination of solutions
reciprocal tying-up of solutions

a model of the production and spatial structure of a regional agrarian-industrial complex;
a model of the location of industry;
a model of the spatial structure of an industrial centre (agglomeration of centres, or zone of continuous development);
a model of the process of organising a complex;
a model of the organising of a complex's infrastructure (transport system, fuel and power supply, building industry, services sphere);
a model of the forming of a complex's labour force;
a model of the formulating of a complex's building plan.

Each model in the group is designed to solve the problems of a certain stage and range of matters. In that connection they differ in structure, level of aggregation, and means of reflecting the conditions of the organisation and functioning of facilities, and of space and time.

In accordance with the approach adopted the group contains enlarged models in which the most basic elements of a region's economy are considered (the spatial structure of its system of complexes) or of a complex as a whole (forecasting of its main parameters), and itemised models in which the separate intracomplex industrial centres or elements of the infrastructure are represented, and, finally, models which, though also intended for detailed investigation, are intended for study of a complex rather than its separate parts. All the models of the group are spatial, i.e. territory is represented in them by an aggregate of territorial taxonomic units rather than by a single point. The sole exception is the model for forecasting the main parameters of a complex.

All the models are still linear and are constructed in continuous and compound formulations. The model for drafting the building plan of a complex will, obviously, be a grid one.

The specific conditions of the organising of individual complexes and the different aims of the solutions necessitate the modifications of the separate models, i.e. enlargement or detailing of the reflection of certain conditions, alteration of the means of representing individual facilities, and so on.

In fact, almost every one of the finished models has several modifications.

The level of treatment and degree of readiness for practical use of the separate models differ; so, too, do the problems of further research.

The diversity of the aims of the research makes for differences in the structure of the models used. The group also includes models of an intersectoral type (for forecasting the main parameters of a complex) and of a production-transport type (for organising the infrastructure). But models specially created for the purpose, i.e. *optimising territorial industrial*, *regional mesomodels*, form the basis of the group. In grouping them under this designation we wanted to stress the characteristics of the object of investigation and to distinguish these mesomodels from the other models. They are primarily *territorial* models, since the main objects in them are the taxonomic units of the country's territorial system, but they are also *production* models, since the basis of a formation of any rank of territorial taxonomic units is taken to be production, and the organisation of all elements of the economy and exploitation of resources are optimised on the condition of meeting the output targets of the specialisation industries. And they are *regional* models of course, because they cover the whole aggregate of the elements of the economy of the area being studied, i.e. economic region, while, by composition of conditions and level of aggregation, they occupy a position midway between regional macromodels and regional micromodels, and so, consequently, are *mesomodels*.

A stage-by-stage (consecutive) account of the parameters of the freedom of solutions was adopted for these models, and an unformalised transition from one stage of the solution to another (from one model to another). Thanks to that we have been able to analyse each solution more deeply, extend the number of conditions and factors considered from stage to stage, check the course of the solution, and construct the separate models consecutively so as to introduce them more quickly into the practice of long-term territorial industrial planning. Each model can function either in the group or separately.

Two types of direct connection and feedback are proposed between the models: namely, (1) transmission of information for the next stage of the solution and co-ordination of the solutions of two neighbouring levels; and (2) exchange of data between models either of one stage or of two consecutive stages. The control problem in co-ordination is to check that the solutions of different levels do not contradict one another and satisfy the common criterion of optimality for the problem as a whole. Information is transferred from stage to stage within the group (from top to bottom, as shown in Diagrams 1.5 and 1.6); some of it is obligatory in the next stage and cannot be altered, and some can be made more precise, as a rule, as a result of detailing the solution. There are also feedback connections between all the models—from bottom to top. The information passing in this direction is mainly about increasing the precision of some of the indices of constraints clarified during more detailed analysis.

The co-ordination problems are of paramount importance on the whole not only because, without solving them, a system cannot be created and the structure of the separate models determined but also because co-ordination of the models facilitates solution of the problems of co-ordinating territorial and sectoral planning, which is also needed in order to convert the models into an important tool for practical pre-planning research.

The stages of the solution. The general problem of optimising the organisation of a territorial industrial complex is solved, as already noted, in several stages, each of which is a logical continuation of the preceding territorial investigations. The number of stages needed and the set of models used are determined by the goal of the investigation, the length of the future period, the specific features of the area being studied, and the degree of coverage of the initial information.

With long-term forecasting of the development of the productive forces of a major economic region, when it is necessary to bring out the features not only of the region as a whole but also of its main parts (complexes) and to determine their specialisation in the inter-regional and intraregional division of labour, the investigation can be limited to Stages I and II, or II and III (see Diagram 1.4)

For medium-term forecasts, when analysis of the spatial organisation of the region's economy and its parts is already called for, a quite detailed investigation of each complex is required, namely, the location of all elements of its economy; the organising of the complementary industries and infrastructure; the direction of exploitation of resources; the complex's external and internal connections; detailed analysis of the forming of the separate parts of each complex (territorial units and elements of the economy) as well as of the complex as a whole—the number of stages will be greater—all five.

The objects, the set of factors considered, and the degree to which they are analysed are determined in accordance with the place of each stage in the pre-planning investigation and the content of its problems.

In Stage I of optimisation all the interconnected complexes of the economic region are investigated; in Stages II and III the separate complexes; in Stage IV the territorial parts of a complex (industrial centres and their agglomerations, and the systems of settlement); and in Stage V the problems of forming the complex in time, and of the development of the separate elements of its economy.

At the same time as the objects of research (the region's system of complexes, and individual complex and its parts) are broken down, the elaboration of the analysis of the economy's elements, and of the factors involved in forming the structure and the character of the spatial organisation of production, and the sequence and rate of development both of the complex's parts and its whole, are increased (see Table 1.2).

In Stage I attention is mainly paid to investigating the location of enterprises of the specialisation industries, allowing very roughly for the specific features of the economies of the separate complexes' areas. Transport, problems of town planning, the power and building bases, the agricultural zones around towns, and local resources are considered solely from the angle of clarifying their effect on the siting of enterprises of the specialisation industries of the whole economic region, and on the organisation of the infrastructure of inter-regional and general regional significance.

In Stage II the investigation is already being made at the level of each separate complex. Not only are the specialisation industries analysed but also the complementary ones. Agriculture, transport, and the services sphere are treated as autonomous elements, but all elements of the economy are considered as a whole for the complex, without analysing their distribution and location.

In Stage III enterprises of the specialisation and complementary industries figure as autonomous planning objects. Transport is represented by the sections both of intercomplex trunk lines and lines of intracomplex significance. The scale of development of the infrastructure and territorial location of its elements are also determined. The conditions for developing the area are studied in detail and also possible variants of the exploitation of a complex's individual subareals and of the mass of farmland, as well as of its major parts or areals (as was done in Stage I).

In Stage IV the same elements of the economy are considered as in Stage III but in greater detail at the level of the zone of continuous development, of agglomerations of industrial centres, and of separate centres. In Stage V it is not so much the objects of the investigation that are elaborated as the period of the forming of the complex.

Three groups of factors have the greatest effect on the organisation and efficiency of large intraregional territorial industrial complexes: namely, natural resources, manpower, and the economic conditions of the complex's formation. The set, the means of taking the separate factors into account, and the degree to which they are elaborated in the various models and at the different stages of the investigation are substantially altered in accordance with the problems and the period of planning.

The differentiation is also the same with different approaches to solving the problems of one and the same stage (e.g. by models of the spatial structure of a complex and of the location of its industries in Stage III; of industrial centres and zones of continuous development in Stage IV; the process of organising a complex and shaping its infrastructure in Stage V). The sole exception is the condition characterising the complex's place in the territorial division of labour (output targets of the specialisation indus-

Table 1.2.

Characteristics of the

Stage	Rank of the territorial industrial combination	Purpose of the investigation	Content of the stage
I	Economic region	clarification of the system and specialisation of complexes	optimisation of the development and location of the facilities of the specialisation industries and elements of the inter-regional and general regional infrastructure
II	Complex	determination of the proportions of the development of the economy's main elements	optimisation of the rates and scale of development in time of interconnected facilities of the specialisation industries and infrastructure, and of the exploitation of resources
III	Complex	determination of the main scheme for allocation of the productive forces	optimisation of the development and location of all elements of the economy and the exploitation of resources

Stages of the Investigation

Models employed	Direct result—proposals by variants for	Form of the pre-planning document — possible use of results
model of the spatial structure of the region's system of complexes	development and location of the specialisation industries and of elements of the infrastructure; location of complexes; specialisation of each complex	general plan for the development and location of the country's productive forces; general development plan for the region's productive forces
model for forecasting the main parameters of a complex	sequence, rates, and scale of development of interconnected facilities and allocation of resources in time	techno-economic substantiation of regional planning schemes; general development plan for the sectors of production of the USSR
model of the spatial structure of a complex; model of the production and spatial structure of a regional agrarian-industrial complex; model of the location of industry	development and allocation of the productive forces; location of industrial centres within a complex; specialisation of industrial centres	general development plan for the region's productive forces; general development plan for the productive forces of a complex

Table 1.2. (*continuation*)

Stage	Rank of the territorial industrial combination	Purpose of the investigation	Content of the stage
IV	Industrial centre	determination of spatial structure	optimisation of the development and location of the elements of the centre's economy and of the exploitation of resources (for agglomerations of centres and zones of continuous development)
V	Complex	determination of the development scheme and building plan	optimisation of all elements of the economy, and of the sequence and periods of building

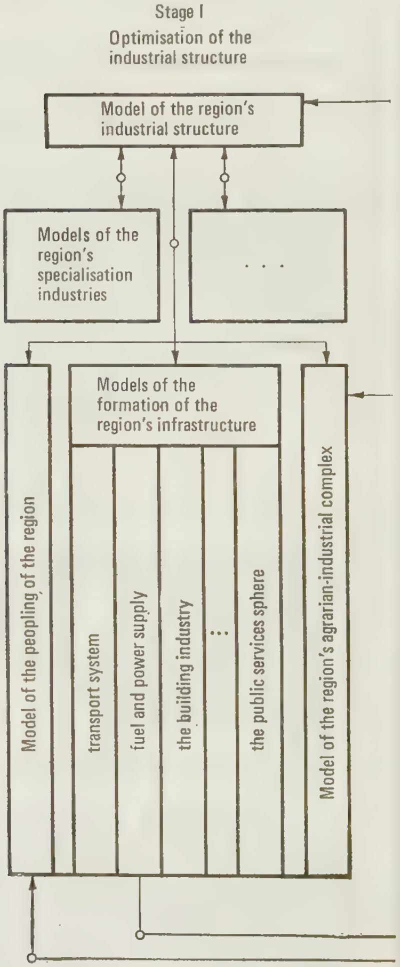
tries), which constitutes an obligatory limitation at all stages and in all the models.

Each stage has its own characteristics, range of matters to be resolved, list of factors and conditions that must be made more precise, degree to which they are to be elaborated, and forms of allowing for them, set of initial information, models and directions of analysis of the results obtained (see Diagram 1.6).

Stage I of the solution thus pursues two aims: namely, (a) *delineation of complexes, and* (b) *determination of the specialisation of each complex.* It is preceded by optimisation

Models employed	Direct result—proposals by variants for	Form of the pre-planning document — possible use of results
model of the spatial structure of an industrial centre (agglomeration or zone of continuous development)	functional zoning of the area; development and location of complementary industries and elements of the infrastructure of centre or intersettlement significance; exploitation of resources; basis of the system of settlement	regional planning schemes; scheme of the region's system of settlement; plan for nature conservation and reproduction of the region's resources; development scheme for the region's individual economic sectors
models of the organising — of a complex; of the infrastructure; of the labour force; model of the building plan of a complex	scale, rates, and periods of development of the individual elements of a complex's economy as a whole	schemes of the general plans of industrial centres

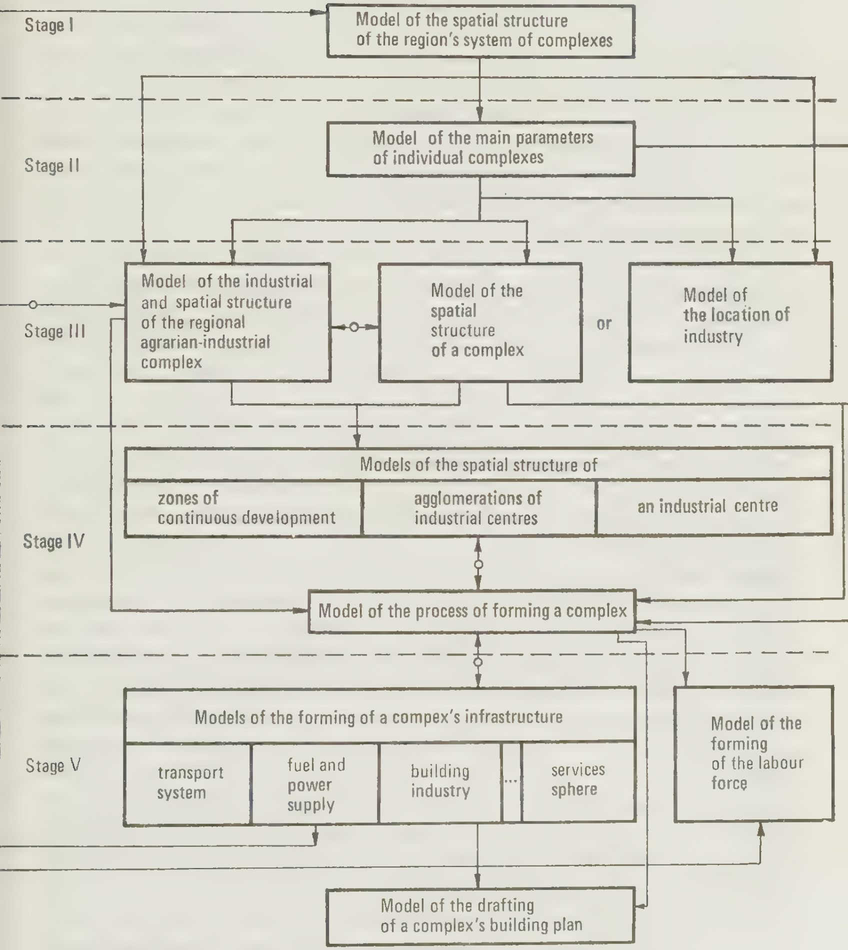
of the industrial structure of the region as a whole and solution of the separate intraregional complex problems (see Diagram 1.6, Stage 1), when the region's specialisation in the inter-regional division of labour is made precise, i.e. the composition and delivery targets of the specialisation industries are made more precise, the targets and the exploitation of resources in short supply are determined, and also the norms for the population's living conditions, and the inputs by which the national economic efficiency of the exploitation of the region's resources has been clarified, since its complexes are closely interlinked. In addition, *Diagram 1.6.* **Optimisation of the**


Stage I
Optimisation of the industrial structure
Model of the region's industrial structure
Models of the region's specialisation industries
. . .
Models of the formation of the region's infrastructure
Model of the peopling of the region
transport system
fuel and power supply
the building industry
...
the public services sphere
Model of the region's agrarian-industrial complex

Structure of a Region's Economy

Stage II

Optimisation of the spatial structure


Stage I
Model of the spatial structure of the region's system of complexes
Stage II
Model of the main parameters of individual complexes
Stage III
Model of the industrial and spatial structure of the regional agrarian-industrial complex
Model of the spatial structure of a complex
or
Model of the location of industry
Stage IV
Models of the spatial structure of
zones of continuous development
agglomerations of industrial centres
an industrial centre
Model of the process of forming a complex
Stage V
Models of the forming of a compex's infrastructure
transport system
fuel and power supply
building industry
...
services sphere
Model of the forming of the labour force
Model of the drafting of a complex's building plan

Conventional symbols as in Diagram 1.5

until the facilities of the specialisation industries and elements of the inter-regional infrastructure, and of the complementary industries closely associated with them, have been sited and until the need to draw on the resources of the separate sectors of the region's territory has been clarified, it is impossible, in practice, to solve the problem at the level of an individual complex. In that connection the conditions for the building and functioning of the facilities of the specialisation industries and of the inter-regional and general regional production infrastructure are analysed, and also the conditions for the exploitation of local resources and for the allocation of resources in short supply over the whole region and within the areals delineated.

The territorial industrial complexes themselves do not come into the problems of Stage I as autonomous territorial units. It is assumed that they are not yet known or that our representation of their future system may be mistaken. The main task of Stage I is, therefore, to delineate the future complexes, determine their specialisation, and make the specialisation of existing complexes more precise.

The initial position adopted for solving the problems of Stage I is that the concentration of facilities of the specialisation industries and of elements of the inter-regional and general regional infrastructure determines the choice and specialisation of areals while the combination of areals and the interareal communications of the elements of the production infrastructure form the skeleton of the spatial structure of a complex. The separate complexes and the whole system of complexes of the economic region are delineated from the results obtained by solving the problem of Stage I on the basis of analysis of the relations between areals.

A model for optimising the spatial structure of a region's system of complexes is proposed for solving the problem ot Stage I. The formulation of the problem and the model are described in detail in Chapter 4.

The following variants are chosen at Stage I of the solution, allowing for both the production and the territorial factors: namely, the connections for obtaining raw materials and for interareal exchange of output; the conditions for providing electricity and fuel, for multipurpose exploitation of the interareal grid, for employing manpower, and for exploiting land and water resources; the conditions of town planning and building, organising the building industry, and creating and operating the local agricultural zone.

From the results of the investigation of Stage I the following are determined: the variant for the allocation of basic resources among the main consumers; the enlarged scheme of the location and connections of production units; the directions, structure, and scale of interareal freight traffic and transmission of electricity; the variant for development of the power grid and of the interareal transport system; the demand for labour; and the volume of capital investment by areals. Analysis of the data obtained makes it possible to determine the areals of concentration of production units and their connections, and at the same time to clarify the composition and specialisation on a national and regional scale of the region's whole system of complexes.

It was found during work on the problems of Stage I of the investigation of complexes that the danger of a too great elaboration, analysis, and aggregation was greater than in the other stages. It is possible to modify the formulation of the problem and model depending on the specific character of the area being studied, the length of the period being considered, and the purpose of the results obtained. In addition, it is more necessary to take the stochastic character of the initial information into account when solving the problems of Stage I than in the other stages.

After allocation of the targets for output of the specialisation industries and of the region's general multipurpose resources among the complexes, and determination of the scale of development and location of the major facilities of the main elements of the general regional production infrastructure, it becomes possible to pass on to investigation of the individual intraregional complexes.

The specialisation of a complex revealed by the results of Stage I characterises its place in the territorial division of the Soviet Union as a whole, and of the region (indicates the composition of the specialisation industries, their scale of development at the end of the plan period, the complex's external connections by deliveries of their output and import of needed resources). However, the solutions of Stage I do not yet provide a full picture of a complex as an integrated, constantly developing system.

The composition and scale of development of the specialisation industries affect the structure and development of the complementary industries and the size of the population. The need for proportional development of the interconnected elements of the economy is reflected in the structure and level of development of the production infrastructure. The increase in population caused by the growth of industry calls for a corresponding development of the elements of the social infrastructure. The scale of the resources that will be needed for proportional development of these elements of the complex's economy is largely determined by the level of development of the complementary industries and of the production and social infrastructure at the beginning of the plan period, and also by the magnitude of the free local reserves of labour. These relations of the elements of the complex's economy are studied in Stage II, at the stage of analysing the proportions of the development of its main elements.

In order to allow more fully for these connections it is necessary to consider the forming of the complex in time, which calls for dynamic information on its external connections. We assume that this will become known through solving the dynamic regional and sectoral problems.

Thus, the *problems of Stage II* of the investigation are formulated as follows: *to determine the parameters of the organisation of a complex in time as a first approximation*, viz. the demand for and allocation of multipurpose resources as a whole; the sequence and rates of building the main facilities of the specialisation industries; the composition, scale and rates of development of the complementary industries and elements of the production and social infrastructure of general regional and complex significance by intervals of the total planning period, given meeting of the targets of the specialisation industries, provision of the stipulated living conditions for the population, observance of the limitations adopted, and minimising of the normalised inputs involved in building the complex.

To solve these matters we propose using a model for forecasting the basic parameters of a complex (Chapter 5), a model that reflects the interconnections of the elements of the complex's economy in time. The general picture of the organisation of the complex's economy with the given specialisation is clarified from the results of the investigations of Stage II. If the analysis indicates that the complex will not be built up with the allotted limit of capital investment, it is necessary to return to Stage I and once again make its specialisation more precise by taking more detailed information on its specific character into account. After that one can pass to solution of the problem of Stage III, and use some of the information obtained to formulate the problem of Stage V (see Diagram 1.6).

In *Stage* III, during determination of the principles of the plan for the development and allocation of the complex's productive forces the possible *variants of the location and functioning of all the main elements of its economy are investigated for each complex*, the network of intracomplex industrial centres is delineated, and the specialisation of each of them clarified. The problem is solved, as in Stages I and II, after the following have been determined: namely, the delivery targets and composition of the specialisation industries; the scale of development of the elements of the intercomplex infrastructure; the limits for the complex on the use of resources in short supply. More detailed data on the conditions for the following are drawn on as initial information than in Stage I: the forming of the population; the development of the separate parts of the complex's area; and the exploitation (utilisation) of all forms of resources. The objects of investigation are also represented in a more differentiated form (see Table 1.1): enterprises rather than aggregated industries; intersub-areal communications rather than interareal ones; sub-areals rather than areals; and the resources of a sub-areal are taken into account by categories for the sectors of its area and not simply for the sub-areal as a whole, and so on. In some cases labour is considered in a sex break-down.

Special attention is paid to analysing the location of industry and agriculture. But the location of their facilities (unlike the sectoral problems for industry or agriculture) is optimised in this case together with determination of a variant of the allocation and exploitation of both single-

and multipurpose resources, and of the scale of development and scheme of location of the main elements of the infrastructure, i.e. resources and services are not allocated before the solution is obtained. The location of production units and their concentration as well as of transport facilities at certain points or groups of points are the basis for the organisation of industrial centres and transport junctions, systems of settlement, the unified transport network, and other elements of a complex. Consequently, when the location scheme for industry is being worked out, the scheme of the spatial structure of all the complex's elements is established, i.e. the basis for the scheme of development and allocation of the productive forces of the complex is formulated.

Two ways of working out the scheme of the spatial structure of the complexes and their parts are possible, depending on the production structure and scale of development of the individual complexes and the aims of the investigation. These ways differ in the manner the objects are represented in the models, in the scope of the analysed elements of the economy, the composition and structure of the models employed, the set of the initial data needed, and the difficulties involved in preparing these data.

One way is employed when an integrated multisectoral complex with a highly developed industry and agriculture, a large set of possible variants of the location of individual production units, and a variety of the conditions of their operation and interrelationships is being investigated. It is impossible, in practice, for such a complex, to consider in advance all probable variants of the location and connections of the individual production units and of the organisation of industrial centres. In that case the plan of the complex's spatial structure is determined by two different but interconnected models: namely, (a) the location of all elements of the economy except the sectors of the agrarian complex is optimised (the model for the spatial structure of a complex in Chapter 6), while agriculture is studied separately with the industries closely connected with it, i.e. the regional agrarian-industrial complex (the model for its industrial and spatial structure is given in Chapter 7). The need for separate solution of the two problems, and for employing different models, is due to the fact that optimisation of the location of industry and agriculture calls for taking many different factors and the specific conditions of each industry into account.

The reciprocal influence and connections of industry and agriculture (competition for the use of land, water, labour, main transport routes, and certain other resources; the need to provide places of concentration of urban population with perishable farm produce; the need to process certain types of farm produce on the spot) are taken into consideration in the form of the appropriate constraints (equations and inequalities) in the models of the spatial structure of a territorial industrial complex and of the industrial and spatial structure of the agrarian-industrial complex. They can be solved in any sequence. The results are mutually adjusted, and after several iterations a reciprocally correlated variant of the location of industry, agriculture, transport, infrastructure, and population becomes clear.

The use of the two models to determine the plan of the spatial structure of an individual complex yields many interesting data for thorough analysis but calls for a great quantity of laborious initial information. These same linear models have two serious inherent drawbacks that have to be taken into account when the results are being analysed. (1) It is impossible to allow during the solution for changes in the techno-economic indices of the individual planning objects with changes in their scale of utilisation. (2) The possibilities of reducing inputs on production, building of the infrastructure, and exploitation of resources through the combinations of production units arising during the solution in the composition of the separate industrial centres are not sufficiently taken into account.

Because there are still no regional non-linear models that satisfy the requirements of the practical problems of allowing for the non-linearity of certain dependences and of the effect of concentration, a second way of solving the problem of determining the spatial structure of a complex's economy has been proposed. It can be employed to investigate complexes with a narrow specialisation of industrial production, a small number of facilities, and poor development of agriculture or market gardening near towns, in which it has been possible to assume the location and probable industrial structure of the industrial centres in advance. In addition, this way can also be employed to study integrated complexes when a detailed analysis of the location scheme for industry is not needed but only a rough (preliminary) analysis.

The problem boils down to determining a variant of the development and location of industry, of the industrial structure of the main industrial centres, and of the development plan of the transport system and system of settlement, given obligatory fulfilment of the production targets of the specialisation industries, observance of the given limitations and minimisation of normalised expenditure on the building and functioning of all the facilities studied.

In order to use the second way of determining the spatial structure of a complex we suggest a model of the location of its industry by centres (see Chapter 8).

With the second approach the means of reflecting a complex in a model is changed. When the first way is used the main objects of optimisation are individual enterprises, the means of representation variants of their functioning, and the unknowns the indices of production, given different variants of location and production connections; but when the second way is employed the main objects become industrial centres, the methods of representation variants of their functioning, and the unknowns locations, the structure of production, the direction and structure of the centres' communications, and variants of the reconstruction of sections of the transport system and system of settlement. The objective function of the solution with either method remains the same, namely, minimum inputs in connection with meeting the production targets of the specialisation industries while observing the limitations adopted.

The basic difference is that, with the first variant, the industrial centres are formed during the solution, while with the second variant they are formed before the solution, during preparation of the initial data. Variants of their formation are differentiated by the set of undertakings, systems of settlement, communication schemes, and development of the infrastructure. Manpower reserves and their composition are taken into account in the process, and also the characteristics of the sectors of the area suitable for building, the state of the water supply, the hygienic and sanitary requirements of the individual production units, and other conditions.

As a result of solving the problems of Stage III of the investigation (employing either two or one model) with a different degree of elaboration, a scheme of the development, location, and connections of all the complex's elements is obtained, i.e. the scale and direction of the exploitation of all types of resource are determined, and also the location of the facilities of the specialisation and complementary industries, of the main facilities of the infrastructure, the location, scale of development, and industrial structure of the industrial centres, the distribution of freight traffic among the different types of transport and the plan of the complex's transport system, the distribution of population and the structure of employment by centres.

As a result of completing the first three stages of optimisation, a general picture of the production activity and spatial structure of each complex of the economic region is obtained. However, there still remain certain unresolved, or insufficiently elaborated problems of the functioning of the separate elements of the economy and territorial parts of each complex. Among these we can put, in the first place, investigation of the forming of certain elements of the infrastructure, viz. the building industry, the services sphere, individual industrial centres, the combination of centres with areals of intensive agricultural production, and the system of settlement. It is also necessary to make a fuller analysis of the possibilities and, above all, ways of providing the complex as a whole and its separate parts with certain types of reproducible, structurally complex resources (e.g. labour).

These matters can only be resolved after the structure of production and the general plan of the spatial organisation of the complex have been clarified. To analyse them it is necessary to allow in more detailed fashion for a large number of specific microconditions, which it is impossible or irrational to do in the preceding stages of the investigation. Their transfer to the final stages makes it possible to relieve as it is complex models of Stage III, and fully corresponds to the sequence of the pre-planning drafting of the long-term schemes for development of the productive forces of the separate areas.

Stage IV of the solution of the general problem of optimising the organisation of a complex, viz., determination of the spatial structure of the industrial centres, is a logical continuation of Stage III. It is done after the network of intracomplex industrial centres has been delineated in the preceding stage, the specialisation of each of the centres determined, and the configuration of the intercentre transport system, and the development variant of the other elements of the infrastructure of significance for the whole complex have also been clarified. The problem of *Stage IV* is *functional zoning of the area, and optimisation of the development and location of all elements of the economy within the centres, of the exploitation of resources, and of settlement.* A model of the spatial structure of an industrial centre is proposed to solve it (see Chapter 9).

Industrial centres and agglomerations of centres, zones of intensive development of agriculture, and even zones of compact development must not be autonomous objects of territorial long-term planning as is the case with a complex. But that does not eliminate the need to consider them as objects not only of the pre-planning investigation of sectors but also of complexes. The fact is that many interdepartmental connections arise within these territorial formations, above all as regards the exploitation of resources and of elements of the infrastructure. And there is always a real possibility of improving these connections and of making a substantial saving of funds in developing the area and all elements of the economy. It is not enough, therefore, to compile schemes of settlement, land use, and other partial solutions; it is necessary to draft unified schemes for the development of these areas and for the spatial organisation of the economy.

To draft them within the context of the group of models in Stage IV, models of the aggregate of industrial centres and zones of continuous development are provided, in addition to the model of the spatial structure of an industrial centre (see Diagrams 1.5 and 1.6).

The zone of continuous development is big enough for solving matters of principle, because it is the basis of a complex, while it is small enough to permit solution of concrete engineering and planning problems.

The results of Stage IV, using one or more models, bring out the spatial structure of the separate parts of a complex. They will be employed to adjust the indices of the problems of Stage III and as initial data for Stage V, when schemes of the system of settlement and of the general plans of the industrial centres, and other pre-planning projecting documents will be worked out.

Most of the models of the group are static; the results of solving them bring out what a complex will be like by the end of the planning period. But that is not enough for many of the pre-planning investigations. One also has to know the process of the complex development. The character of the development of the separate parts of the area and the sequence for building the facilities are essential so as to choose between one variant and another; and substantial reduction of the input of social labour can be achieved through rational development of both the separate elements of the complex and of their combinations. In that connection, after a quite detailed analysis of the complex as a whole and of its respective parts (industrial centres and agglomerations) *Stage* V is tackled, i.e. analysis of the process of organising the complex as a whole and its separate parts, and *determination of the scheme for the organisation of the complex.*

In essence Stage V consists in determining a variant of the forming of the complex as a whole (in time), i.e. the complementary industries, infrastructure, exploitation of resources, and settlement, allowing (a) for the targets for development of the facilities of the specialisation industries and elements of the intercomplex infrastructure and provision of living conditions for the population, and (b) for the location plan of the main facilities of the specialisation industries and the sequence for developing the separate parts of the area clarified by the results of the preceding stages of the investigation. A dynamic model of the forming of a complex is proposed for solving this problem (see Diagrams 1.5 and 1.6) a detailed description of which is given in Chapter 10.

Optimisation of the organisation of a complex calls for solving many problems of the forming of the infrastructure. Each infrastructural sector and many of its separate parts have specific features that must be taken into account when the variants for the development, location, and connections of the facilities are being determined. In general the development and location of all types of infrastructure are considered at all stages of the solution, and the infrastructure is represented with a different level of itemisation in all the models of Stages I to IV.

In spite of the fact that the development scheme of the infrastructure, and of its composition, connections, and location, has already been solved in general outline and made precise through optimising the formation of the complex, it is necessary, in Stage V, to solve problems for each sector of the infrastructure and for certain multipurpose resources (e.g. manpower). In that connection the group of models includes models of the organising of the complex's infrastructure and labour force (see Diagram 1.5). These problems cannot, however, be finally solved at complex level because they are linked with the organising not simply of one complex but of the respective sectors or of the population in the whole system of the region's complexes. This explains the reciprocal tying-up envisaged in Diagram 1.6 of the results of the solution obtained from these models with the results of the solutions from the model of the forming of a complex and from the models of the respective intraregional, complex programmes at regional level.

Stage V and the whole work are crowned by the drafting of the building plan for the complex. Until a model has been constructed for that purpose, drafting of the investment programme for the complex will obviously remain the final stage of the solution.

During the five stages of the investigation it is proposed to solve three types of problem differing in content and calling for a special structure of the mathematical economic models and the principles for constructing them.

1. *Determination of the optimum variant of the spatial structure of each complex* (*Stages I*, *III*, *IV*). This problem can be formulated in general form as follows: determine the variant of the exploitation of resources, location of indus-

tries and elements of the infrastructure of the respective complexes, given fulfilment of their role in the territorial division of labour and minimisation of the normalised inputs on their building and functioning. A series of optimisation models is proposed for this problem, corresponding to each rank of territorial industrial combination: namely, for an economic region, a model of the optimisation of the spatial structure of its system of complexes; for an individual complex, a model of its spatial structure and a model either of the production and spatial structure of the agrarian-industrial complex or of the location of industry; for part of a complex, models of the spatial structure of the economy of the zone of continuous development and of an individual industrial centre or agglomeration of centres (see Diagrams 1.5 and 1.6). The following are considered simultaneously in each of these models: the industrial units, infrastructure, and resources of the complex concerned.

2. *Analysis of the formation of separate complexes* (*Stages II and V*). The essence of this group of problems consists in verifying the correspondence of the sequence, periods, and rates of building of the facilities of the specialisation industries, the scale, periods, and rates of development of the complementary industries and elements of the infrastructure and of the exploitation of resources, on condition of minimising inputs on the building of the complex.

To resolve this type of problem dynamic optimisation models for forecasting a complex's main parameters and the process of its forming are proposed. The possible variants of the organisation of the specialisation industries are represented in them; and freedom is permitted in formulating the variants of the development of complementary industries, all the elements of the infrastructure, and the exploitation of resources. A joint, interconnected survey of all production units and all elements of the infrastructure is the great achievement of these models, because it renders it possible to represent the formation of the complex under study as a whole.

3. *Determination of the variant of the shaping of the separate elements of the infrastructure and of the building plan of a complex* (*Stage V*). The content of the problems of this type boils down to elaborating variants of the location,

connections, rates of development, and structure of each type of infrastructure of the complex concerned, given minimisation of the normalised inputs on the building and functioning not only of the complex's whole economy but also of the given type of infrastructure on the scale of the whole region. Separate analysis of the development of each individual type of infrastructure (transport system, building industry, fuel and power supply), and not of the whole aggregate of them, is the main difference in principle between the problems of Stages II and III. In the given case the interconnections of each separate type of infrastructure and the whole economy are studied, and as a result, possibilities develop not only of obtaining a detailed scheme of the organisation of the type of infrastructure concerned but also of adjusting the input indices according to it.

However perfect the methods of mathematical modelling, it is impossible in practice to reflect fully and reliably in a model the whole diversity of the factors and conditions affecting the development of a real economic system. Any optimal plan obtained from computations on models must, therefore, be adjusted in accordance with the concrete conditions of their realisation.

When the group of models for optimising the forming of complexes at Stage I is employed, the specialisation of the complexes is determined by means of a model of the spatial structure of the economic region. The solution from this model serves, as we have already noted, as the initial data for models of the spatial structure of each of the separate complexes (Stage III), functioning, moreover, as firm assignments. The field of possible solutions of the problem of the spatial structure of a complex is consequently already partially limited by the computations of Stage I. On the other hand, the initial indices characterising an individual complex in the model of Stage I [the indices of the exploitation of local resources (land, water, etc.), the building and functioning of industrial facilities, the plan for the location of which is optimised in the Stage I model] undergo changes after solution of the problems at Stage III. These alterations are not only linked with the possibility of a more detailed taking account, at Stage III, of the conditions of each separate complex but are also connected with optimising the location of the given aggregate of industries in definite sub-areals. In these conditions the need to determine the field of possible adjustment of the optimum solution of the problem becomes an obvious insufficiency of a single optimum solution of the problem of a higher level for determining the variant of the specialisation of the individual complex.

By *field of adjustment we mean the aggregate of intervals of alteration of the problem's economic indices, each of which has its own corresponding structure of a plan that is optimal only within this interval.* Obtaining the field of adjustment gives a chance of clarifying the set of proposed strategies of the functioning of the system being studied corresponding to the changes in the initial indices for the separate elements of the system within these intervals. Clarification of the aggregate of strategies thus makes it possible to determine not only the direction of the adjustment of the optimum solution of the problem but also its qualitative expression, corresponding to the changes in the economic indices.

During practical determination of the field of adjustment several difficulties arise in the forming of the interval of change of each economic index and in simulating the behaviour of the latter within this interval and the interaction of the economic indices (allowing for the simultaneity of their possible changes). To solve this problem we used a method of investigating the zone of indeterminacy of an optimum solution developed at the Power Institute of the Siberian Department of the USSR Academy of Sciences.* Determination of the field of adjustment includes several stages each of which has a corresponding complex of programs for an electronic computer (see Diagram 1.7).

To begin with, an initial variant of the problem is calculated. Econometric analysis of the solution obtained brings out the factors substantially affecting the formulation of the optimum plan, special attention being paid to the factors in which changes in the initial data can cause a change in the structure of the optimum plan. For each of these

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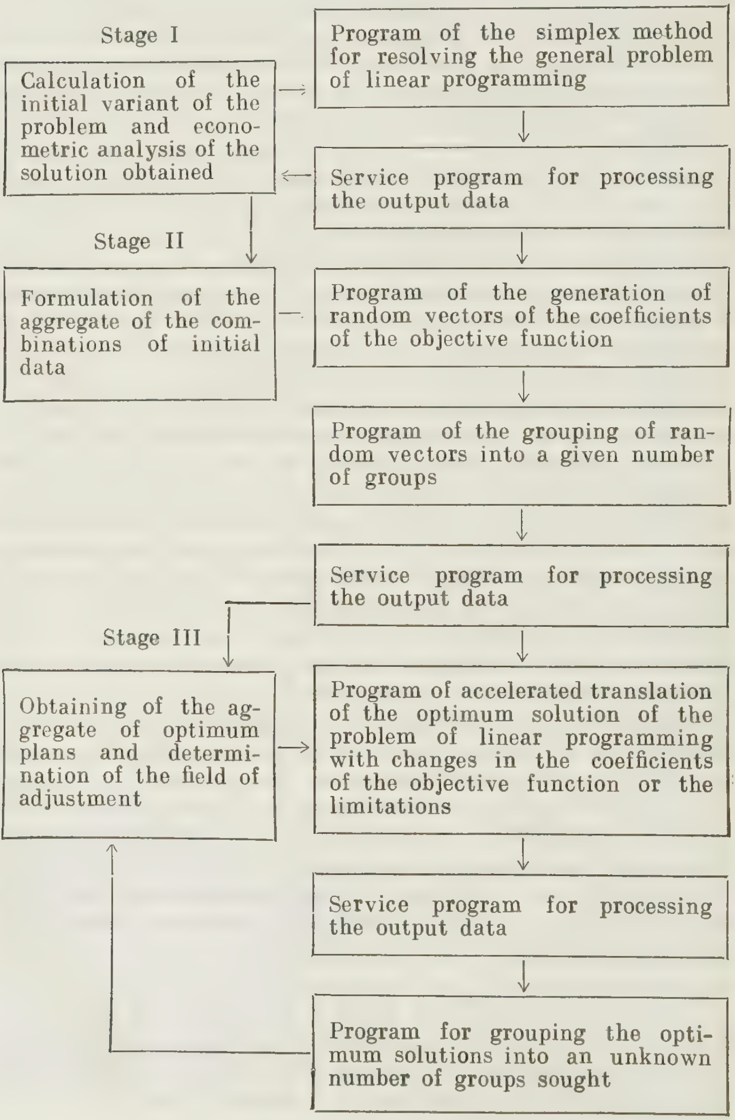
* A. A. Makarov, "Basic Ideas of the Method of Studying the Zone of Indeterminacy of the Optimum Solution of Linearised Systems". In *Algoritmy i programmy issledovaniya zony neopredelennosti optimalnogo resheniya* (Algorithms and Programmes for Studying the Uncertainty Zone of an Optimum Solution), Irkutsk, 1970, pp. 4-14.

Diagram 1.7. **Main Stages in Determining the Field of Adjustment and Set of Computer Programs**


Stage I
Calculation of the initial variant of the problem and econometric analysis of the solution obtained
Program of the simplex method for resolving the general problem of linear programming
Service program for processing the output data
Stage II
Formulation of the aggregate of the combinations of initial data
Program of the generation of random vectors of the coefficients of the objective function
Program of the grouping of random vectors into a given number of groups
Service program for processing the output data
Stage III
Obtaining of the aggregate of optimum plans and determination of the field of adjustment
Program of accelerated translation of the optimum solution of the problem of linear programming with changes in the coefficients of the objective function or the limitations
Service program for processing the output data
Program for grouping the optimum solutions into an unknown number of groups sought

factors the upper and lower limits of the intervals of change of their indices are determined from expert economic appraisals.

The possible combinations of the altered initial indices are then calculated, the mathematical apparatus for which makes it possible to take a multiplicity of interconnected factors into account affecting the change in each index, the simultaneity of random changes in the various indices, and the behaviour of each index within the limits of the accepted interval of change. Each of the combinations obtained forms a new variant of the problem differing from the original one in the quantitative values of the initial economic indices.

Finally, an aggregate of optimal plans reflecting the behaviour of the economic system with changes in its initial information within the given intervals is determined. The solutions obtained are grouped according to the uniformity of their structure, the number of groups not being known in advance. The intervals of change in the economic indices of the problem, for each of which there is a corresponding structure of the optimal plan, are clarified, i.e. the field of adjustment of the optimum solution is determined.

The results of the experimental computations made indicated the expediency of determining the field of possible adjustment of the optimum solution of the problem of forming a complex. During formulation of the intervals of change in inputs constituting the field of adjustment, it becomes possible to represent and evaluate the behaviour of the studied economic system in more detail, and to determine the degree of the effect of the various factors taken into account in the problem on the choice of the optimum variant of the location scheme of the industries. This method can probably be employed to co-ordinate the system of models.

* * *

The proposed approach to employing mathematical economic models to optimise the development of territorial industrial complexes, and the sequence of the solutions, are not universal; the models are standard ones, i.e. are suitable for investigating any region and any complex in the Soviet Union. Territorial industrial complexes are very diverse; even within a large economic region (e.g. Siberia) they differ in their place in the economy of the USSR, in the time they come into being, in the scale and level of development of the productive forces, in the composition and extent of the resources, in the industrial and spatial structure of the economy, and in the conditions of their organisation and functioning. In that connection there cannot be standard models that would equally satisfy the requirements for optimising the formation of any complex. Changes must be introduced into them in the writing of certain conditions, depending on the characteristics and the set of the most pressing problems of the complex being studied.

2

COMPOSITION OF THE INITIAL INFORMATION

V. D. Ionova and M. A. Malinovskaya

The composition of the initial information to solve the problems of optimising the formation of territorial industrial complexes is determined by the economic posing of the problems, the characteristics of the mathematical economic models, and the nature of the criterion adopted. The initial information includes the following:

(a) indices of the limitations defining the field of acceptance of solutions;

(b) coefficients of the matrices characterising the main technical and economic indices of the objects and constraints of the problems;

(c) coefficients of the objective function reflecting inputs by variants and modes of functioning of the objects;

(d) the intracomplex conditions for the building and functioning of the elements of the economy for each territorial taxonomic unit (region, complex, industrial centre).

The formulation of information for solving the problems by the group of models proposed here rests on the principle of stage-by-stage detailing and supplementing of the initial data. The information put into a model at one stage or another is an assignment or target from the solutions of the preceding stage, but is also supplemented by indices detailing the sectoral and territorial aspect. The content and character of the initial data are altered depending on the purpose of the investigation (see Table 2.1).

All the initial information is thus divided into two types, as follows:

1. Indices obtained through the solution of problems of a higher stage or taken from the data of project studies and qualitatively analysed. They include the basic indices forming the limitations; the composition and capacities of plants of the specialisation industries, determined from the solution of the problems in Stage I, for example, reflect the assignment for the structure of the complex's production (Stages II and III).

Table 2.1.

Initial Information for the Group of Models for Optimising the Organisation of a Territorial Industrial Complex

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
A. Production Sphere							
I. Industry							
1. Facilities of the specialisation industries with variants:							
of location	+	—	+	+	+	—	+
of combination	+	+	+	+	+	+	+
of production links	+	+	+	+	+	—	+
of period and intensity of building	—	+	—	—	—	—	+
of capacity	+	+	+	+	+	—	+
of technological schemes	—	—	+	+	—	—	+
2. Facilities of the complementary auxiliary industries	—	+	+	+	—	+	+
3. Output targets of the specialisation industries	+	+	+	+	+	—	+
4. Demand for raw materials, supplies, fuel and power (indices of the input of the given resources per unit of the main product)	+	+	+	+	+	+	+

Table 2.1. (*continuation*)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
5. Demand for labour (indices of the total and unit labour cost by plants of the specialisation and complementary industries), total	+	+	+	+	+	+	+
men and women separately	—	+	+	+	+	+	+
interchangeability	—	+	—	+	—	+	+
6. Demand for capital investment (including building and construction), total	+	+	+	+	+	+	+
by periods of construction	—	+	—	—	—	—	+
7. Operational inputs	+	+	+	+	+	+	+
8. Inputs on transportation (indices of normalised unit transport costs)	+	—	+	+	+	+	+
9. Need for land siting facilities	+	—	+	+	—	+	+
for creating sanitary protection zones	+	—	+	+	—	+	+
10. Demand for water							
water supply, total,	+	—	+	—	—	+	+
including single-flow	+	—	+	—	—	+	+
discharge facilities for industrial effluents, total	+	—	+	—	—	+	+
including those requiring special forms of treatment	+	—	+	—	—	+	+

Table 2.1. (continuation)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
II. Agriculture							
1. Production targets for the main types of farm produce	—	+	+	+	+	—	+
2. Agricultural zones and their specialisation	—	—	—	—	+	—	—
3. Demand for capital investment (including building and construction) by types of production and agricultural zones	—	—	—	—	+	—	—
4. Current inputs by types of production	—	+	—	—	+	—	—
5. Demand for labour, total	—	+	—	—	+	—	+
male	—	+	—	—	+	—	+
female	—	+	—	—	+	—	+
6. Agricultural demand for industrial types of raw and other materials, and for fuel and power	—	+	—	—	+	—	+
B. Resources							
I. Raw Materials							
1. Composition, location, and variants for developing sources of raw materials and fuel	+	+	+	+	—	—	+

Table 2.1. (*continuation*)

Composition of the initial information	Models: of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
2. Normalised inputs by variants of the development of each source of raw material resources	+	+	+	+	—	—	+
3. Shipment targets, by type of resource	+	+	+	+	—	—	+
4. Limit on imports, by type of resource	+	+	+	+	—	—	+
5. Normalised transport inputs, by type of resource and type of transport	+	—	+	+	—	—	+
II. Manpower							
1. Population, numbers	+	+	+	+	+	+	+
2. Total able-bodied (working) population:	+	+	+	+	+	+	+
male	—	+	+	+	+	+	+
female	—	+	+	+	+	+	+
3. Employment structure of the able-bodied population by economic sectors within a complex,	—	+	+	+	+	+	+
including by the periods of development	—	+	—	—	—	—	+
4. Natural and mechanical increments of population and their structure, total	+	+	+	+	+	+	+
by periods	—	+	—	—	—	—	+

Table 2.1. (*continuation*)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
5. The population's demand for local and outside resources	+	+	+	+	+	+	+
6. Limit on the influx of labour	+	+	+	+	—	—	+
III. Local resources (land, water)							
1. Taxonomic units treated as objects of investigation in the given models (areals, sub-areals, sectors, centres)	+	+	+	+	+	+	+
2. Characteristics of units from the standpoint of readiness for industrial and civil building and agricultural production	+	—	+	+	+	+	+
3. Variants of the development of each taxonomic unit of the area	+	—	+	+	+	+	+
4. Inputs by variants of development (normalised inputs per unit of developed area)	+	—	+	+	+	+	+
5. Schedule of agricultural zones	—	—	—	—	+	—	—

Table 2.1. (*continuation*)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
6. Water sources of each taxonomic unit; types, volume, variants for regulating flow	+	−	+	−	−	+	+
7. Cost of water-engineering works by variants of the use of water sources	+	−	+	−	−	+	+
C. Infrastructure							
I. Production infrastructure Transport							
1. Scheme of the system of all types of transport within separate areas (region, complex, industrial centre)	+	−	+	+	+	+	+
2. Carrying capacity of sections and junctions of the transport system, variants of their building and development	+	−	+	+	−	+	+
3. Demand for capital investment to reconstruct and build transport facilities	+	−	+	+	−	+	+

Table 2.1. (continuation)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
4. Volume of traffic at junctions and on sections of the transport system (of separate areas): incoming, outgoing, transit, and local freight	+	—	+	+	—	+	+
5. Demand of transport for local resources and products of other sectors of the economy	—	+	—	—	—	—	—
Electricity and heat							
1. Composition and techno-economic indices of the facilities of the power grid and variants of its development	+	—	—	—	—	—	—
2. Scheme of the local power system and variants of its development	—	—	+	—	—	+	+
3. Limit on drawing electricity from the grid	+	+	+	—	—	—	+
4. Target for feeding electric power into the system	+	+	+	—	—	—	+
5. Variants of the power supply for facilities at the end of the period by periods	+ —	— +	+ —	+ —	+ —	+ —	+ +

Table 2.1. (continuation)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
6. Variants of heating supply for facilities	−	−	+	+	−	+	+
7. Normalised inputs involved in generating and transmitting:							
electricity	+	+	+	+	−	−	+
heat	−	+	+	+	−	−	+
Building industry							
1. Facilities of the building industry by variants of							
location	+	−	−	+	−	−	+
capacity	+	+	−	+	−	−	+
zones serviced	+	−	+	+	−	−	−
2. Demand for local resources and products of other sectors	+	+	−	+	−	−	+
3. Inputs on creating the building industry (normalised inputs, per unit capacity)	+	+	+	+	−	−	+
Municipal engineering structures and works							
1. Maximum possible capacity of facilities of the intracentre infrastructure	−	−	−	−	−	+	−
2. Throughout of underground mains and service	−	−	−	−	−	+	−

Table 2.1. (continuation)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
3. Maximum possible rate of operating of underground mains and services	—	—	—	—	—	+	—
4. Labour inputs on operating engineering works	—	—	—	—	—	+	—
5. Capital inputs on developing sections of underground mains, etc.	—	—	—	—	—	+	—
II. Social infrastructure							
1. Inputs on providing services for the population (indices of normalised unit outlay on foundations and profile drawing for civil building per 1000 persons), differentiated within the limits of taxonomic units of the area							
(a) by areals	+	—	—	—	—	—	—
(b) by sub-areals	—	—	+	+	—	—	+
(c) by sectors	—	—	—	—	—	+	—
2. Inputs on building the complex of facilities of the town social and public infrastructure, separately by sub-areals and sectors (indices of the normalised unit inputs per 1000 persons):							
total for the whole ac-							

Table 2.1. (continuation)

Composition of the initial information	Models						
	of the spatial structure of the region's system of complexes	for forecasting the main parameters of a complex	of the spatial structure of a complex	of the location of a complex's industry	of the forming of the regional agrarian complex	of the spatial structure of an industrial centre	of the process of organising a complex
1	2	3	4	5	6	7	8
counting period	—	—	+	—	—	+	—
separately by periods	—	+	—	—	—	—	+
3. Composition, capacities, and purpose of facilities of the social infrastructure by sub-areals of a complex at the beginning of the period under review	—	+	+	+	—	+	+
4. Number of jobs in the non-productive sphere (indices of the ratio of jobs in the non-productive sphere to workers in the town-determinant group of industries)	—	+	+	+	—	+	+

Conventional symbols: + signifies initial data taken into account in formulating the problem;
— signifies that data have not been taken into account.

2. Indices calculated directly at the stage of preparing the initial data for the concrete problem. They include mainly indices forming the coefficients of the matrices and the objective function, e.g. calculation of the inputs needed to build and site concrete enterprises arising from the local siting conditions.

Productive sphere. The following initial data are required to determine the spatial structure of the system of territorial industrial complexes:

1. The national economy's need for the products of the region's specialisation industries: namely, the composition and capacities of the specialisation industries, and the character and scale of their external production links; the availability of local resources of various kind to guarantee operation of these plants; and data on the limit on resources from outside.

2. The state of and outlook for the development of existing industrial facilities in the region's economy; the scale and direction of their production links with newly built plants and the possible scale and direction of external inter-regional production links.

3. The availability and scale of the region's natural resources and character of intraregional production links for clarifying variants of the organisation of large production units of the complementary industries, which it is necessary to built, in turn, for normal operation of the plants of the specialisation industries.

4. The location of existing and possible areals of the concentration of new plants of the specialisation industries within the economic region, and the main techno-economic indices reflecting the capital and current costs needed to build and operate the plants in one area or another, depending on its local conditions.

In Stage II of the investigation the initial information for determining the main parameters of a complex is obtained from the solution of problems of Stage I, and is supplemented and detailed from the data of the project planning bodies. Thus the targets for output of the main products of the specialisation industries are obtained from the model of the spatial structure of the region's system of complexes, while the project studies by specialisation industries make it possible to make the schedule of enterprises, their projected capacities, and main techno-economic indices more precise (cost of production of the main products; capital investment on new building and reconstruction; standard indices of inputs of power, raw materials, supplies, and labour).

During preparation of the initial information for solving a given problem possible variants of the development of enterprises are constructed, taking account of the different periods for starting the building of plants and of the rate at which their capacities will increase. Variants of the development of a complex are thus also differentiated by production links, depending on whether local resources and products are used, or are brought into the complex from outside.

At Stage II of the investigation the rough scale of development of the complementary industries is determined. Their probable capacities and their techno-economic indices are obtained from the project planning data.

When matters of the spatial structure of a complex are being decided (Stage III), the interconnections of industries are considered in more detail (volume of freight traffic for raw and other materials, and finished products of the enterprises of the specialisation and complementary industries), and also the effect of the complex's local conditions: namely, the demands made by enterprises on the area (space required; need for water; sanitation conditions, etc.). These data are calculated by standard indices, taking into account the concrete circumstances of the location of the complex's enterprises over its area.

During the preparation of the initial information variants of the location of the plants of the specialisation industries are drawn up by sub-areals of the complex. The opportunities for siting plants in sub-areals, and of forming industrial centres, depend to some extent on the availability and scale of local resources, the inputs on exploiting them, and the requirements of environment conservation. At the same time the variants drawn up are also differentiated by their intrasectoral and intersectoral production links.

The target for volume of output by enterprises of the specialisation industries is obtained from the answers to the problem provided by the model of the spatial structure of the region's system of complexes, and is decided for the complementary industries by the answers obtained on the demand of plants of the specialisation industries. The techno-economic indices of the inputs of raw materials, supplies, power, and water, and of the area needed for building plants and for hygienic-sanitary zones are calculated from average sectoral norms and standards, taking concrete conditions into account, or from the data for each complex supplied by sectoral research project planning institutes.

At Stage III the problem of optimising the development of agriculture is also solved. The initial information for this problem includes detailed data on the microconditions of each complex and data on plants processing agricultural raw materials, their techno-economic indices, and variants of their location, depending on the raw material base and concentration of population.

At Stage IV the initial information on production comes from the answers of Stage III, and includes the output targets of the plants of the specialisation and complementary industries, the main technological links between the separate enterprises of the industrial centre, the assignments for import and export of raw materials and output, the techno-economic indices of exploitation of the centre's local resources (land and water), and the numbers employed in the productive sphere.

At this stage information is prepared by variants of the joint exploitation by the centre's plants auxiliary and service industries, and of its engineering services and local resources.

Natural resources, manpower, and other resources. The initial information characterising this group of indices is also presented with different degrees of itemisation at each stage of the investigation.

At Stage I the data on the resources of the area, being considered are reflected in a summed or aggregated form. Mineral resources, for instance, are represented by the major deposits, or even basins, that need to be developed so as to create the region's specialisation industries; power resources according to the capacity of the major generating stations and the opportunity of obtaining power from the grid, and so on. The conditions for developing the area are reflected through inputs, differentiated by areals.

At Stages II and III resources of raw materials, fuel and power are considered independently of whether they are located within the complex or outside it. The maximum possible volumes of extraction of resources from local sources are defined, allowing for the reserves of minerals and the conditions for mining, processing, and employing them. At the same time, account is taken of the magnitude of the demand for these resources in other regions of the country. The limits on deliveries of resources from outside a complex are given by the answers of Stage I.

The size of a complex's labour force is limited and is found from the data of the statistical agencies, allowing for the long-term outlook of natural and mechanical growth of the population. At the same time the sex structure of the labour force is considered at Stages III and IV. This is necessary so as to determine the rational employment of men and women in the productive and non-productive sphere. At all stages of the problems being resolved not only is the need for labour taken into account, but the total population is also calculated for the individual taxonomic units through the demographic load factor, allowing for employment of some of the population in the infrastructure sectors and the non-productive sphere.

The size of the area operates as a limitation at all stages. At Stage I it is the areal, which embraces several sub-areals for industrial and civil building; at Stage III the size of the concrete sub-areals for building separate industrial centres; at Stage IV, the size of the sectors set aside for industrial and civil building, i.e. the division into industrial and urban zones.

The inputs on developing possible sub-areals are calculated from their characteristics (size, relief, soils, availability of water, conditions for the discharge of effluent, etc.): namely, the inputs on the initial cycle of building, surface levelling-out compensating for farmland used for other purposes, and so on.

Stage IV estimates the complex's resources which are to meet the needs of not only industry but also the non-productive sphere, the needs of which were averaged through coefficients at higher levels of the system. For the sectors of a centre, for instance, data are given on their location, area, configuration, conditions of exploitation, and so on. The total balance of the water resources of an areal, calculated at Stage I, is supplemented at Stage III by indices of the water intake for each water source supplying a definite sub-areal. The water consumption regimes arising from specific features of the technology of a certain plant, and the regimes for regulating the annual and seasonal flow of rivers, etc., are also considered.

Elements of the infrastructure. The elements of the infrastructure include transport, power supply, the building industry, municipal engineering works, and the service industries.

At Stage I the initial information on transport consists of the characteristics of the economic region's transport system (namely, existing, projected, and probable sections of the system of all types of transport), the possibility of joint use of the various types of transport is also being assessed. During preparation of the initial information the indices of transport costs are also decided, i.e. the total inputs on carrying freight by sections of the region's transport system and the input on capital construction by variants of reconstruction and new building.

At Stage II, the initial information on transport reflects the indices of transport calls on individual complex's resources, so as to determine the main parameters of its development. Stage III necessitates more detailed data on transport, because the development of the transport system of a separate complex is optimised at that stage. Information on the complex's external transport links is taken from the problems of Stages I and II. During preparation of the information for Stage III the complex's transport system is broken down into the sections between its probable industrial centres. Information is prepared for each section on its carrying capacity; the volume of actual freight in both directions is established, and variants for the freight flows within the complex are calculated, including the building of new sections of the transport system.

At Stage IV the system of sidings and access lines within an industrial centre is assessed. The flow of commuter rides within the centre is determined, for which variants of the carrying capacity of possible access lines are calculated, plus variants of their technical equipping and variants of their freight traffic over a period of time. The time and normalised inputs on the labour's commuter rides are calculated by variants.

The initial information on the supply of power to plants of the specialisation industries is represented at Stage I by indices reflecting the magnitude of the limits and direction of the distribution of power from the region's unified grid. This includes the volumes of power consumption of plants of the specialisation industries, and of the mandatory supply and inputs associated with the transmission of power and the building of major transmission lines. During the preparation of the initial information the indices of the use of power by plants of the specialisation industries are formed by variants of their connection to one power source or another. These indices are also taken into account at Stage II, when the interconnections of power sources and the rest of industry are allowed for through standard norms of power consumption by the plants of the complex's industry, population, and other elements of its economy.

At Stage III the initial techno-economic information from project planning and research organisations is supplemented by data from the optimisation solutions on possible power sources (arising from local needs) and their dovetailing with the separate industrial centres. Variants of the supply of power and heat to the complex's enterprises from the central and local sources are drawn up on the basis of indices of the inputs in both physical and value terms.

The third element of the infrastructure is the building industry. When the production specialisation of a complex is being determined, the initial data describe the existing and possible capacities of building facilities, their needs for multipurpose resources, and the inputs on creating the industry, differentiated by the various parts of the region, and also the demand of the objects of planning for the services of the building industry.

At Stage II and IV the building industry is considered in greater detail. Its capacity is determined by periods of the building of the complex. For that purpose variants are calculated for the functioning of the industry by periods (the volume of building and construction work), the demand for building materials (output of the complex's own building industry plus materials brought in from outside) and the complex's other resources (electric power, water, land);

the number of jobs in the building industry is also established by periods.

The initial information on the social infrastructure for the model of the spatial structure of the system of complexes (Stage I) is represented by the average aggregated value inputs on providing services for the labour force. At Stage II value inputs on housing and amenities are summated by periods. In the model of the spatial structure of a complex these inputs are differentiated for its areal, depending on the proposed variants of the location of enterprises and plants, and of the population settlement. In the model of the spatial structure of an industrial centre, building and the municipal economy (power supply, district heating, sewerage, and so on) are considered separately. The value indices of the inputs on building the infrastructure are differentiated by the area of the centre in accordance with the variants of settlement (the building of one town or several housing estates for a group of enterprises).

At Stage V the problem of the organisation of a complex is resolved, for which the question of the location of complementary industries and other facilities of the economy is decided at the same time as the dynamics of the development of the specialisation industries. Together with the data needed to answer the Stage II and III problems of a complex, more detailed consideration is given to data pertaining both to the inputs that the complex must allocate and in order to build enterprises of the specialisation and complementary industries and of the production and social infrastructure over its whole area, and of the inputs associated with the need to attract and settle labour and provide the population with farm produce. In particular, manpower is investigated in detail, and the employment and interchangeability of male and female labour in industries determined.

3

DIRECTIONS OF THE ANALYSIS OF THE SOLUTIONS OF SPATIAL STRUCTURE

V. S. Zverev

Analysis of the answers obtained on spatial structure is an essential stage in developing the plan for allocation of the productive forces of the area being studied. Its direction is determined by the specific character of the problems considered, the main purpose of which is to ensure proportion between the different elements (factors) of the area's economy.

Analysis of the effect of factors on the spatial structure of an area's economy as a whole, and on each other, necessitates employing, in the first place, such methods of mathematical economic analysis as analysis of the shadow prices* and their components obtained from the inverse matrix of the optimum solution. For special purposes (analysis of separate modes of operation of enterprises and plants, and facilities of the infrastructure, and of drawing on resources) other methods can also be used. It is possible to employ shadow prices, in particular, to substantiate the use of new modes of operation of facilities not taken into account during the solution, and how far modes not covered by the optimum plan are close to it.

From the results of the analysis we can work out the directions of subsequent improvement of the spatial structure of the area's economy.

The solutions of spatial structure are analysed in the following order (see Table 3.1):

(a) general analysis of the spatial structure of the area's economy;

(b) analysis of the economy's external connections (targets, limits on delivery of resources, etc.);

(c) clarification of the specific features of the location and operation of the individual elements (facilities) of the economy within an area;

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* See L. V. Kantorovich, *Ekonomichesky raschet nailuchshego ispolzovaniya resursov* (Economic Calculation of the Optimum Use of Resources), USSR Academy of Sciences Publishers, Moscow, 1959.

Table 3.1.

Directions and Content of the Analysis of an Economy's Spatial Structure

Line of analysis	Aim of the analysis	Indices employed	Application of the results
General analysis of the economy's spatial structure	to determine the possibility of siting facilities within separate territorial units and of exploiting their resources;	indices of the demand of facilities for resources and their availability by territorial units	selection of facilities and means of exploiting resources, by territorial units
	to determine the specialisation of the territorial units and the exploitation of their resources	indices of the composition of production units and of the exploitation and availability of resources by territorial units	clarification of the lines of future exploitation of resources, by territorial units
Analysis of external connections	to determine the effect of external connections on the size of the objective function	shadow prices of targets of the specialisation industries and of the limitations on external resources	directions of adjusting targets by specialisation industries and limits on external resources
Clarification of the factors of the location of the economy's elements	to determine the effect of external and internal conditions on the location and operation of the facilities of the specialisation industries, infrastructure, etc.	components of the shadow prices of the constraints for the relevant facilities	clarification of ways of improving the efficiency of the location and operation of the facilities being studied

Table 3.1. (*continuation*)

Line of analysis	Aim of the analysis	Indices employed	Application of the results
Clarification of the factors of the organisation of the economy of territorial units	to determine the effect of the combination of the plants of the specialisation industries, facilities of the infrastructure, and local resources on the organisation of a territorial economy	components of the shadow prices of the constraints for facilities of the specialisation industries located in a given territorial unit	clarification of ways of improving the efficiency of the organisation of the territorial units' economies
Joint analysis of the separate elements of the economy at all stages of optimisation of its spatial structure	to determine the role of the individual element in the organising of the spatial structure of the economy of a region, complex, or industrial centre	components of the shadow prices of the constraints for facilities of the specialisation industries	clarification of the formulation of the problem of the different stages of optimising the spatial structure of a region's economy

(d) study of the specific features of individual sites (areals, and parts of areals).

General analysis of the spatial structure of an area's economy helps bring out the main outlines of the plan for the location of the productive forces. It is done at the stage of formulating the problem and after obtaining the quantified solution.

When the problem is being formulated the possibility of including separate variants of the location and functioning of facilities of the economy (or of their combinations) in the plan is analysed, and also the possibility of employing separate sources of resources or all the resources of separate sites.

The initial information for this analysis is presented in the form of a table for each site (see Table 3.2)

Table 3.2.

Availability and Possible Lines of Exploitation of the Resources of a Site

Types of local resources	Unit of measurement	Magnitude of resources by sources, ranked by the size of the index of the objective function				Demand of facilities (plants, etc.) located within the area		
		1	2	...	total	1	2	...
1	2	3	4	5	6	7	8	9
Area	hectare							
...	...							

Comparison of the quantity written in column 6 of Table 3.2 with the corresponding index in each of the next columns makes it possible to discover the facilities that can be located at the given location, i.e. the facilities for which the demand for all forms of local resources is less than their availability at that location.

One must bear in mind that, with close links between facilities, the possibility of their joint (group) location at a given point is considered and their demand must be summated, allowing for the effect of joint location. In that way we indicate the demand for local resources by facilities presented by the aggregated means.

When, however, there is no single facility (or combination of facilities) that can be located at a given point, this point is omitted from the number of possible sites.

Given the availability of several units that could be located at a point, comparison of their total demand (the sum of the values of columns 7, 8, etc. of Table 3.2) with the availability of resources by separate sources (i.e. with the values in columns 3, 4, and 5), arranged in order of magnitude of the indices of the objective function of the problem, makes it possible to determine which sources of resources can be left out of account. In that way, at this stage of the general analysis of the scheme of the spatial structure of the area's economy, the areals (sub-areals, and parts of sub-areals) and the sources of resources within them that it is expedient to consider during the numerical solution of the problem are delineated.

When the numerical solution (of the scheme of the economy's spatial structure obtained) is being analysed we determine the character of the area's development, the scale of the resources being developed, and the conditions for implementing the given development programme of the specialisation industries, delineate the areas of compact and focal development and the reserve areals (sub-areals, and parts of sub-areals) for future development of the economy, the sectoral structure of production, and the specialisation of the different complexes (industrial centres).

Table 3.3.

Exploitation of the Local Resources of a Site

Types of local resources by sources	Unit of measurement	Magnitude of the resources, by sources	Exploitation of resources by located facilities, by sources	Reserves of resources, by sources
1	2	3	4	5

Note: the table is compiled for each site or point of location.

The characteristics of the resources of the separate location points are given by comparison of the availability and exploitation of local resources (see Table 3.3). Comparison of the quantities in columns 3 and 4 of this table for each location point indicates which types and sources of resource are in short supply or are lacking, and which still have reserves. The data obtained can serve as the basis for developing proposals for the location of a definite profile of production units at the points considered in the next stages of the development of the area's economy.

In the course of the general analysis of the spatial structure of an area's economy it is most important to compare the solutions of problems that differ (within certain limits) in the parameters of the constraints and techno-economic indices of the ways of building and operating the facilities, i.e. of the results of the variant calculations.

It is important, for the problems of optimising the spatial structure of an area's economy, to trace the effect on the location of the productive forces of changes in the targets of the specialisation industries, the limitations on use of the area's resources, and indices of the objective function (adjusted costs).

Comparison of the results of variant calculations that take into account the probabilistic character of the parameters of the problem and the possible error in individual parameters makes it possible to delineate the stable nucleus (the basis of the plan) of the spatial structure of the economy of the area being studied.

In the other lines of analysis the employment of special indices is envisaged, i.e. of shadow prices (prices of the optimum plan and of the dual variables), and of their components (the constituents of the shadow prices). Shadow prices represent the solution of the dual problem. And since the direct and dual problems are solved simultaneously (in one computation process), the shadow prices are obtained as part of this solution. Shadow prices reflect the relationship between the constraints and the objective function in the direct problem and are of the same economic nature as the indices of the objective function.

With changes in the problem's constraints, when the structure of the ways of locating and operating the facilities of the area's economy that come into the optimum plan (the basis of the optimum plan) remains unaltered, the shadow prices do not change in their value. For the problems of spatial structure being considered, stability of the shadow prices is observed within quite wide limits of changes in the constraints of the problem. That is so because the separate sources of the area's local resources have quite broad possibilities of meeting additional demand for a resource without altering the techno-economic indices for drawing on (or obtaining) it.

Shadow prices' property of relative stability makes it possible to evaluate the effect of certain changes in the problem's constraints on the magnitude of the objective function, and to clarify the direction for adjusting the plan obtained.

The constraints that in the aggregate form the problem of optimising the spatial structure of an area's economy are varied both in structure and in their application to the various elements of the region's economy (or that of a complex or industrial centre). The whole variety of these constraints, however, can be reduced to a few main types, as follows.

1. Limitations on the rate of operation of the plants of the specialisation and complementary industries, and facilities of the production infrastructure (linear and point), and the existing facilities of the social amenities' infrastructure, on the employment of local labour and use of the local resources of individual sources, and on bringing products and resources in from outside the given area.

The shadow prices of these limitations indicate how far the total input on the building and functioning of the area's economy can be reduced (increased) with an increase (reduction) in the rate of exploitation of a given production unit or source of resources in the accepted unit of measurement.

Because most of the given constraints in the model take the form $\sum_{r \in R} x^r \leqslant 1$ (where x^r is the rate of utilisation of a capacity (resource) in the rth variant, and R is the set of the variants of the facility's functioning, a change in the limitations by unity would mean exceeding the limits of stability of the system of shadow prices, in that case, therefore, the analysis employs unit shadow prices characterising a change in the limitations by a quite small value; as a rule, the unit of measurement of the product of the given facility serves this purpose;

2. The input-output conditions of production and consumption of the product, by facilities of the specialisation and complementary industries; the existence and utilisation of output capacity, by units of the linear infrastructure; the production and utilisation of output (services), by facilities of the point infrastructure; and the availability and employment of labour and local resources, by territorial units.

The shadow prices of these conditions indicate how much the outlays on the building and functioning of the area's economy can be increased (reduced) with an increase (reduction) in the demand for corresponding products (services) or resources by unity.

3. The targets for producing output of the specialisation industries or for shipping it out of the area by the different directions.

The shadow prices of production targets indicate how much total outlays on the building and functioning of the area's economy can be increased (reduced) with increase (reduction) of the given target by unity. When these constraints take the form $\sum_{r \in R} x^r \geqslant 1$, then, as with the limitations, unit shadow prices are employed (per unit of output corresponding to the given constraint).

The shadow prices of targets for shipping output indicate how the total inputs on the building and functioning of the area's economy can be increased (reduced) with increase (reduction) of the shipment target through the boundary point concerned by unity.

A very important way of using shadow prices is to discover possible changes in the constraints of the problem for obtaining a more efficient structure of the area's economy (from the aspect of the criterion adopted).

This applies primarily to the constraints reflecting the economy's external links; namely, the production (shipping) targets for the specialisation industries and limitations on the bringing of products and resources from outside the area.

The value of the shadow prices indicates how far the separate constraints affect the magnitude of the objective function. The greater the absolute value of the shadow prices the more significant is the effect of the given constraint on the total of the inputs on the building and functioning of the area's economy. The existence of a zero shadow price is evidence of the absence of any effect by the constraint concerned on the total of the inputs.

Since each of the constraints corresponds to a certain type of output of the specialisation industries or of product (resource) brought in from outside, their shadow prices characterise the relative efficiency of the separate specialisation industries or bringing in separate types of product (resource), within the context of the given structure of the economy. Comparison of the shadow prices of the given constraints therefore makes it possible to discover the direction for adjusting the parameters of the production targets and of the limitations on importing output (resources), and to work out proposals for altering the parameters of the operation of the industries in the problems of the preceding stages of the investigation.

The specific features of the location and operation of the individual elements (facilities) of the economy of a region (complex, industrial centre) are manifested through the effect of the sectoral and intersectoral conditions and other factors of the area concerned on each of the elements of its economy. The tools for analysing this are the indices of the breakdown of the shadow prices of the targets, limitations, and input-output matrices of the production and distribution of the product.

When the components are comparable in absolute magnitude or exceed the shadow prices (which is possible in view of the different signs of the components), one may conclude that the effect of the factors concerned on the location and operation of the given element of the economy (facility) within the area may be unstable if there are components of different signs substantially greater than the absolute value of the shadow prices or stable when the separate components are of the same order as the shadow prices.

The specialisation industries are analysed by comparing the components of the shadow prices (the indices of columns 3 to 10 in Table 3.4) with the shadow prices and with one another.

The components of the shadow prices presented in Table 3.4 reflect the total effect of the aggregated factors. If a

Table 3.4.

Structure of the Shadow Prices of the Constraints for Facilities of the Specialisation Industries

Content of constraints	Shadow prices of constraints	Components of a shadow price corresponding to the aggregate of modes							
		of locating and operating				of employing resources		of effecting connections	
		given industry	other industries	the infrastructure		labour	other	external	internal
				production	social-amenity				
1	2	3	4	5	6	7	8	9	10

Table 3.5.

Structure of the Shadow Prices of the Constraints for Facilities of the Complementary Industries and Infrastructure and for Sources of Resources and Connections

Facilities, plants, sources of resources, communications	Shadow prices, the constraints of the location and functioning of facilities, and use of resources	Components of the shadow prices corresponding to the modes														
		of locating and operating							of using resources				of making connections			
		separate enterprises (plants) of the specialisation industries			complementary industries and the infrastructure				of the source concerned	of other sources			of the connection concerned	of other links		
		1	2	. . .	of the unit concerned	of other units				1	2	. . .		1	2	. . .
						1	2	. . .								
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

more detailed analysis is wanted of the influence of separate elements of the production infrastructure, manpower, and local resources, and of separate industries and links on the location and operation of facilities, then those components of the shadow prices are considered that correspond to the ways the separate elements of the infrastructure and industries are located and operate, and the ways resources are drawn on (by types) and connections realised. The values of the components of the shadow prices are used to establish not only the effect of some one factor on the location and functioning of facilities within the given area but also its relative size. For that purpose factors are ranked by the values of the components of the shadow prices. The factor with the strongest effect is taken to be that with the largest value of the component (in absolute terms) of the shadow price.

The facilities of the complementary industries and infrastructure, and sources of resources call for more detailed analysis of their reciprocal effect on each other and of the effect on their development (exploitation) of the facilities of the specialisation industries. For that purpose the shadow prices of their constraints are broken down as shown in Table 3.5. During the analysis of each facility entered in column 1, the value of its shadow price (column 2) is compared with the respective components of the shadow price for the given facility (columns 6, 10, or 14) and with the components of this shadow price for other facilities (columns 3 to 5, 7 to 9, 11 to 13, and 15 to 17). As a result, the relationship between the constraints of the studied facility itself and of the other facilities of the area's economy is brought out.

In addition, the components of the shadow prices for the facilities concerned (columns 6, 10, and 14) are compared with the other components of the given prices so as to bring out the factors decisively affecting inclusion of the facilities concerned in the plan.

Analysis of the specific features of the location and operation of the separate elements (facilities) of the area's economy provides the basis for delineating the specialisation industries whose location and operation are mainly determined by the intrasectoral and intersectoral connections, conditions, and level of development of the infrastructure, by resources of various kinds, and by combinations of the factors listed above. This can be used to draft a regional policy for the development of the separate specialisation industries of its economy. The factors determining the scale of development and location of the complementary industries and infrastructure, the exploitation of resources, and the intensity of a communications route are determined in a similar way, while comparison of the structures of the factors for these facilities makes it possible to determine whether or not their effect is reciprocal or unilateral.

Study of the specific features of individual sites consists in establishing the effect for each of them of (a) joint use by several facilities of different types, sources of resources, and elements of the infrastructure, (b) the operational conditions of the infrastructure and of exploitation of resources, and (c) limitation of the number of sites for certain facilities of the specialisation industries. The following are employed in the analysis: components of the shadow prices of the constraints (the output targets for the product, the input-output matrices of its production and distribution, etc.), by facilities of the specialisation industries located in accordance with the answers for the point (site) concerned (see Table 3.6).

The effect of locating enterprises together in one place can be judged by the specific weight of the index of column 3 in Table 3.6 in the total value of the shadow prices of the constraints (column 2); the effect of facilities of the infrastructure can be gauged by the specific weight of the components corresponding to the variants of their functioning (column 5); the effect of resources—the importance of the components by modes of employing them (column 6). The effect of an industry with a limited number of possible sites within the area concerned is determined through the components corresponding to its modes in the shadow prices of the constraints for all other industries (column 4). The role of each of these factors is determined by comparing the corresponding components in the grand total of the shadow prices of each of the constraints considered. The analysis permits the role of the factors mentioned in creating the structure of the economy of a point to be determined as well as the role of its separate factors. For that the sum of

Table 3.6.

Structure of the Shadow Prices of the Constraints Corresponding to the Location of Facilities of the Specialisation Industries Located at a Point

Objects and content of the constraints	Shadow prices of the constraints	Components of the shadow prices corresponding to the modes				
		of functioning of			of exploiting resources	of function of other facilities
		jointly sited enterprises (plants)	enterprises with a restricted number of possible sites	facilities of the infrastructure		
1	2	3	4	5	6	7

Note: tables are compiled for each site of facilities of the specialisation industries.

the indices of columns 3, 5, and 6 (Table 3.6) are compared with the value of the shadow prices (column 2) and the indices of columns 4 and 7. At those points for which this total is greater than the indices of columns 4 and 7, the local conditions will prove to have the greatest influence on specialisation of the economy.

The results of the analysis bring out the role of the separate sites within the given area, their resources and conditions that are significant for the whole region (complex, industrial centre), and the main factors determining the structure of the economies of the sites, all of which makes it possible to approach the creation of a plan (scheme) of the spatial structure of the area's economy in a more substantiated way.

The solutions of the problems of spatial structure are analysed according to this scheme for the whole economic region (complex, industrial centre). To get an integral picture of the economy of an economic region and of its separate elements, it is necessary to make a joint analysis of the solution of the problems of all three stages vertically and of Stages II and III horizontally (and correspondingly by complexes and industrial centres). This is done by means of the indices of Table 3.7. The role of each element of the economy (transport, power generation, building industry, etc.) in the creation of its spatial structure is brought out at all stages through the development conditions of the facilities of the specialisation industries, because they are the main connecting link of the area's economy both horizontally and vertically.

Table 3.7.

Importance of the Components Relating to an Individual Element of an Area's Economy in the Shadow Prices of the Constraints of the Facilities of the Specialisation Industries at the Various Stages of Optimising the Economy's Spatial Structure

Facilities of the specialisation industries and constraints	Stage I (economic region)	Stage II (individual complexes)			Stage III (individual industrial centres)		
		1	2	...	1	2	...
1	2	3	4	5	6	7	8

By comparing the indices of Table 3.7 we bring out, above all, the distribution of the effect of a given element on the structure of the area's economy. For that the effect of the element is considered as regards (a) stages (region, complex, industrial centre) and (b) territorial industrial units (complexes in a region, industrial centres in a complex). During the analysis the stages are assessed at which the effect of the element concerned makes itself felt in the organising of the area's economy, and also the extent of this effect. The fact and degree of this influence are determined by comparing the indices in column 2 of Table 3.7 with those of columns 3 to 5 and columns 6 to 8, and by comparing the indices of columns 3 to 5 and 6 to 8 with one another. Comparison of the indices of one stage of the solution (columns 3 to 5 or 6 to 8) makes it possible to analyse how far a given element of the economy affects the organisation of various territorial industrial units of the same level.

The analysis also makes it possible to determine the effect of the economy's separate elements on each other as well as on the location and functioning of the plants within the area.

On the whole, the aspects of the analysis considered here determine the conditions and factors shaping the spatial structure of the economy of an economic region, complexes, and industrial centres.

Part II SET OF MATHEMATICAL ECONOMIC MODELS OF OPTIMISING THE FORMATION OF A TERRITORIAL INDUSTRIAL COMPLEX

4

MODEL OF THE SPATIAL STRUCTURE OF AN ECONOMIC REGION'S SYSTEM OF COMPLEXES

M.K. Bandman and V. V. Vorobyova

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

The spatial structure of an economic region is determined after the national economic and general regional problems have been solved, when its place in the country's territorial divisions of labour is known (i.e. the composition and scale of the specialisation industries), the limits established for resources in short supply that the region can obtain from outside sources, the production targets for the output of specialisation industries set in accordance with the local resources and the resources allocated to the region, and the industrial structure of the region as a whole has been defined.

The problem is formulated as follows: *to determine the variant of the location of facilities of the region's specialisation and complementary industries and of the elements of the inter-regional and general regional production infrastructure such as will minimise the total adjusted costs of the building and functioning of the region's economy, with due allowance for the planned standard of living of the population.*

The basic territorial unit employed is the areal, and the main functional units of the region's economy are the aggregated production activities or large plants of the specialisation and complementary industries and facilities of the inter-regional and general regional infrastructure, trunk sections of the region's transport, manpower by catchment zones of areals, and local resources by types and areals.

Production-transport links are represented by flows of import and export of the region's output and by interareal freights. In order to take this into account, border and interior areals are distinguished. External links in and out of the region are effected through border areals.

Solution of the problem posed necessitates allowing for the following factors:

(a) the region's place in the territorial division of labour;
(b) the specific features of the individual production activities;
(c) the natural conditions and resources of the region's separate areals;
(d) the degree to which the area has been developed;
(e) the labour supply of the separate areals, and the conditions for attracting and retaining labour.

The effect of these factors is taken into account through limitations (output targets for the specialisation industries; limits on the possible use of different types of resource, etc.); technological coefficients (indices of the consumption of raw material, fuel, auxiliary materials, output of other production, and local resources); and the coefficients of the objective function (inputs for production by variants of its organisation, depending on the conditions of location, connections, and operation).

The proposed formulation of the problem is formalised by the mathematical economic model presented below, for which the following notation is used.

INDICES AND SETS

i —product or resource index ($i \in I$, $I = I_1 \cup I_2 \cup I_3$);

I_1 —sub-set of the output of the specialisation industries;

I_2 —sub-set of the output of complementary industries;

I_3 —sub-set of types of raw material;

j —index of a production activity or enterprise ($j \in J$, $J = J_1 \cup J_2 \cup J_3$);

J_1 —sub-set of plants of the specialisation industries;

J_2 —sub-set of complementary production activities;

J_3 —sub-set of raw material sources;

k —areal index $(k \in K,\ K = K_1 \cup K_2)$;

K_1 —sub-set of border areals;

K_2 —sub-set of interior areals;

ν —index of the building industry $(\nu \in \Upsilon)$;

Υ —set of the region's building industry;

φ —index of a power system $(\varphi \in \Phi)$;

Φ —set of the power systems of the region's power grid;

m —index of a power station $(m \in Q,\ Q = Q_1 \cup Q_2)$;

Q_1 —sub-set of hydroelectric generating stations;

Q_2 —sub-set of the other power sources in a system;

t —index of the working regime of the generating and consuming elements of a power system ($t = 1, 2$, where 1 is the peak load and 2 is the basis load);

$(\varphi\varphi')$ —index of the transmission line linking power systems φ and φ' $(\varphi, \varphi' \in \Phi$ and $\varphi \neq \varphi'$, $(\varphi\varphi')$ being the direct direction of the line and $(\varphi'\varphi)$ its reverse direction;

(kk') —index of a section of the trunk transport system linking neighbouring areals k and k' ($k, k' \in K$ and $k \neq k'$, (kk') being the direct direction of the section and $(k'k)$ its return direction);

r —index of a variant of the operation of production units or of the exploitation of resources $(r \in R,\ R = R_1 \cup R_2 \cup R_3)$;

R_1 —sub-set of the variants of the creation and functioning of the building facilities;

R —sub-set of the variants of the development and functioning of the sections of the trunk transport system;

R_3 —sub-set of the variant of the exploitation of water sources in an areal.

In order to formalise the links of areals with power systems and the building industry, the set of all areals is represented in the first case as $K = \bigcup_{\varphi \in \Phi} \bar{K}_\varphi$, and in the other as $K = \bigcup_{\nu \in Y} \tilde{K}_\nu$ where $\bar{K}_\varphi$ is the sub-set of areals united in a single power system φ, and $\tilde{K}_\nu$ is the sub-set of areals served by a single construction firm ν.

COEFFICIENTS OF THE MATRIX

A_{ijk} —output volume of product i of plant j in areal k;

B_{ijk} —volume of consumption of product i by plant j in areal k;

f_{jt} —index of the electrical capacity needed for plant j under operation conditions t;

$Э_j\ (Э_{j2})$ —total (basis) power needed for operation of plant j;

Π_{jk} —volume of building and assembly work needed to build plant j in areal k;

T_j —manpower needed to operate plant j;

s_j —land needed for plant j;

w_j —water resources needed for operation of plant j;

P_ν^r —volume of building and assembly work performed by firm ν under variant r of its operation, $r \in R_1$;

$\bar{f}$ —unit coefficient of power capacity used for building and assembly work;

$\bar{Э}$ —unit coefficient of power consumption for building work;

$\tilde{T}_k$ —unit labour intensiveness of building and construction in areal k;

s —unit expenditure of land on creating the building industry;

f —unit coefficient of the public's generating capacity;

π —unit coefficient of the public's consumption of electricity;

P_k —unit coefficient of the utilisation of building work to provide services for the population in areal k;

$\bar{s}$ —unit use of land to provide services for the population;

w —unit coefficient of the public's consumption of water;

γ —coefficient of the ratio of population to the number of workers employed in the production activities and infrastructure being considered;

$\Pi^r_{(kk')}$ —operation capacity of section (kk') under variant r of its operation, $r \in R_2$;

$\hat{T}^r_{(kk')}$ —manpower needed to ensure variant r of the operation of section (kk') of the trunk transport system;

$f^r_{(kk')t}$ —index of the electric power needed to operate section (kk') of the trunk transport system under variant r with regime t of power consumption;

$\text{Э}^r_{(kk')}$ $(\text{Э}^r_{(kk')2})$ —amount of total (base) load needed to operate section (kk') of the trunk transport system under variant r;

W^r_k —water supply, given variant r of exploitation of the water sources of areal k;

$\bar{v}_{\varphi\varphi'}$ —unit transmission loss of power in transmission line $\varphi\varphi'$;

h_t —annual number of consumption hours of electric power, generated by a power source;

$\bar{h}_m$ —coefficient of the ratio of peak and base generating loads for new hydroelectric stations, $m \in Q_1$;

h —coefficient of the current-carrying capacity of a transmission line in terms of capacity and electric power.

CONSTRAINTS

B_{ik} —target for shipping product i out of the region from areal k ($i \in I_1$, $k \in K_1$);

d_{ik} —maximum possible volume of deliveries of product (resource) i from outside the region to areal k ($i \in I_2 \cup I_3$, $k \in K_1$);

T_k^* —numbers of local manpower in the catchment zone of areal k;

S_k —area of land resources in areal k;

$N_{m\varphi}$ —maximum possible generating capacity of power station m in power system φ ($m \in Q_1$);

$M_{m\varphi}$ —maximum possible generation of electricity by power station m in power system φ ($m \in Q_1$);

$\overline{N}_{\varphi\varphi'}$ —maximum possible capacity of transmission line $\varphi\varphi'$;

$\overline{M}_{\varphi\varphi'}$ —maximum possible carrying capacity of transmission line $\varphi\varphi'$;

N_t —power capacity needed for production and infrastructure facilities not included in the model under regime t of power consumption;

E —base power load needed for production and infrastructure facilities not included in the model;

$\tilde{N}_\varphi$ —power capacity needed for production and infrastructure facilities not included in the model, and located in power system φ;

E_φ —total power needed for production and infrastructure facilities not included in the model, located in φ power system;

$\Pi_{(kk')}$, $\overline{\Pi}_{(k'k)}$ —volume of transit traffic on sections (kk') and $(k'k)$.

INDICES OF THE OBJECTIVE FUNCTION

c_{jk} —normalised inputs on building and operating plant j in areal k;

$\bar{c}_{ik}$ —unit normalised input on producing output i outside the region and transporting it to areal k ($i \in I_2 \cup I_3$, $k \in K_1$);

$c_{ikk'}$, $\overline{c}'_{ik'k}$ —unit normalised input on transporting product i from areal k to areal k' and from areal k' to areal k;

$\tilde{c}_{\nu k}$ —unit normalised inputs on the work of building firm ν in areal k;

$\hat{c}^{r}_{\nu}$ —normalised inputs to create and operate building firm ν under variant r $(r \in R_1)$;

c_k —unit normalised inputs on drawing local labour into production and providing it with services in areal k;

$\overline{c}_k$ —unit normalised inputs on attracting labour to areal k, settling it there, and providing it with services;

$\hat{c}^{r}_{(kk')}$ —normalised inputs on providing for variant r of the development and operation of section (kk') of the transport system $(r \in R_2)$;

$\tilde{c}_{m\varphi t}$ —unit normalised inputs on building and operating the capacity of power station m in power system φ, to meet regime t of power consumption;

c —unit inputs on extra supplies of electric power by transmission lines;

$\overline{c}$ —unit inputs of the transmission of electricity by transmission lines;

$c^{\text{new}}_{(\varphi\varphi')}$ —unit normalised inputs on building a new high-voltage transmission line $(\varphi\varphi')$;

c^{ind}_k —unit normalised inputs on developing the land resources of areal k for industrial purposes;

c^{civ}_k —unit normalised inputs on developing the land resources of areal k for civil engineering;

c^{w}_k —unit normalised inputs on supplying water to the production facilities in areal k;

$\overline{c}^{\text{w}}_k$ —unit normalised inputs on supplying water for the public in areal k;

$c^{r\text{w}}_k$ —normalised inputs on providing areal k with water by variant r for the exploitation of water resources $(r \in R_3)$.

UNKNOWNS

x_{jk} —intensity of production activity j in areal k;

$\bar{x}_{ik}$ —volume of deliveries of product (resource) i from outside the region to areal k ($i \in I_2 \cup I_3$, $k \in K_1$);

$x_{ikk'}$, $\bar{x}_{ik'k}$ —volume of deliveries of product i from areal k to areal k', and vice versa;

$z_{\nu k}$ —volume of building work performed by firm ν in areal k;

$\bar{z}_{\nu}^{r}$ —rate of operation of building firm ν by variant r;

x_k —numbers of local manpower drawn into production in areal k;

$\bar{x}_k$ —numbers of manpower attracted to areal k for production needs;

$z_{kk'}^{r}$ —rate of variant r of the operation of section kk' of the interareal transport network;

$V_{m\varphi t}$ —capacity of power station m of power system φ to meet power consumption regime t;

$V_{\varphi\varphi'}$, $\bar{V}_{\varphi'\varphi}$ —volume of the total transmission of power supplied from system φ to system φ', and vice versa;

$u_{\varphi\varphi'}$, $\bar{u}_{\varphi'\varphi}$ —volume of the total transmission of electricity supplied from system φ to system φ', and vice versa;

$U_{\varphi\varphi'}$ —capacity of transmission line $\varphi\varphi'$;

u_k^{ind} —land used for industrial development in areal k;

u_k^{civ} —land used for civil construction in areal k;

u_k —quantity of water needed for industrial purposes in areal k;

$\bar{u}_k$ —quantity of water needed by the population in areal k;

u_k^{r} —rate of utilisation of water sources in areal k under variant r ($r \in R_3$).

The mathematical model of the problem of the spatial structure of an economic region's system of complexes is as follows: namely, find the non-negative values of the variables x_{jk}, $\bar{x}_{ik}$, $x_{ikk'}$, $\bar{x}_{ik'k}$, $z_{\nu k}$, $\bar{z}^r_{\nu}$, x_k, $\bar{x}_k$, $z^r_{kk'}$, $V_{m\varphi t}$, $V_{\varphi\varphi'}$, $\bar{V}_{\varphi'\varphi}$, $U_{\varphi\varphi'}$, $u_{\varphi\varphi'}$, $\bar{u}_{\varphi'\varphi}$, u_k^{ind}, u_k^{civ}, $\bar{u}_k$, u_k^r, that satisfy the following constraints.

The targets for shipping output of the specialisation industries out of the region must be met by areals through which external connections are effected:

$$\sum_{j\in J_1} A_{ijk}x_{jk} - \sum_{\substack{j \\ \text{with } i\in I_2\cup I_3}} B_{ijk}x_{jk} + \sum_{k'} \bar{x}_{ik'k} - \sum_{k'} x_{ikk'} = B_{ik}$$

$$(i\in I_1,\ k\in K_1). \tag{4.1}$$

For interior areals the input-output matrices of the production and distribution of output turned out by the facilities of the specialisation industries must be balanced:

$$\sum_{j\in J_1} A_{ijk}x_{jk} - \sum_{\substack{j \\ \text{with } i\in I_2\cup I_3}} B_{ijk}x_{jk} + \sum_{k'} \bar{x}_{ik'k} - \sum_{k'} x_{ikk'} = 0$$

$$(i\in I_1,\ k\in K_2). \tag{4.2}$$

For areals through which external connections are effected the input-output matrices for the production and distribution of output of the complementary industries must be balanced:

$$\sum_{j\in J_2, J_3} A_{ijk}x_{jk} - \sum_{\substack{j \\ \text{with } i\in I_1\cup I_2 \\ \text{or } i\in I_1\cup I_3}} B_{ijk}x_{jk} +$$

$$+\sum_{k'} \bar{x}_{ik'k} - \sum_{k'} x_{ikk'} + \bar{x}_{ik} = 0$$

$$(i\in I_2\cup I_3,\ k\in K_1). \tag{4.3}$$

For interior areals the input-output matrices of the production and distribution of output of the complementary industries must be balanced:

$$\sum_{i\in J_2, J_3} A_{ijk}x_{jk} - \sum_{\substack{j \\ \text{with } i\in I_1\cup I_2 \\ \text{or } i\in I_1\cup I_3}} B_{ijk}x_{jk} + \sum_{k'} \bar{x}_{ik'k} - \sum_{k'} x_{ik'k} = 0$$

$$(i\in I_2\cup I_3,\ k\in K_2). \tag{4.4}$$

The volume of deliveries of products of the complementary industries from outside the region must not exceed the established limits:

$$\bar{x}_{ik} \leqslant d_{ik} \quad (i \in I_2 \cup I_3,\ k \in K_1). \tag{4.5}$$

For facilities concerned with producing raw materials, the rate of operation must not exceed the maximum possible capacity:

$$x_{jk} \leqslant 1 \quad (j \in J_3,\ k \in K). \tag{4.6}$$

The demand for building work in each areal must be satisfied:

$$\sum_{\nu} z_{\nu k} - \sum_{j} \Pi_{jk} x_{jk} - \gamma P_k (x_k + \bar{x}_k) \geqslant 0 \quad (k \in K). \tag{4.7}$$

The choice of operation variant for each building industry is determined in accordance with the volume of work to be done:

$$\sum_{r \in R_1} P^r_{\nu} \bar{z}^r_{\nu} - \sum_{k \in \tilde{K}_{\nu}} z_{\nu k} \geqslant 0 \quad (\nu \in Y). \tag{4.8}$$

Only one variant can be chosen for the development of each building industry:

$$\sum_{r \in R_1} \bar{z}^r_{\nu} \leqslant 1 \quad \text{with} \quad \bar{z}^r_{\nu} = \begin{cases} 0 \\ 1 \end{cases} (\nu \in Y). \tag{4.9}$$

The capacity of each section of the interareal transport system must be adequate to carry the intraregional and transit freight traffic:

$$\sum_{r \in R_2} \Pi^r_{(kk')} z^r_{(kk')} - \sum_{i} x_{ikk'} \geqslant \Pi_{(kk')} \quad \text{(for all } (kk')) \tag{4.10}$$

$$\sum_{r \in R_2} \Pi^r_{(kk')} z^r_{(kk')} - \sum_{i} \bar{x}_{ik'k} \geqslant \bar{\Pi}_{(k'k)} \quad \text{(for all } (k'k)) \tag{4.11}$$

Only one variant can be selected for the development of each section of the interareal transport system:

$$\sum_{r \in R_2} z^r_{(kk')} \leqslant 1 \quad \text{with} \quad z^r_{(kk')} = \begin{cases} 0 \\ 1 \end{cases} \quad \text{(for all } (kk')) \tag{4.12}$$

The conditions for providing labour for the industries' trunk transport system, and building industry of each areal must be met:

$$x_k+\overline{x}_k-\sum_j T_j x_{jk}-\sum_\nu \widetilde{T}_k z_{\nu k}- \\ -\sum_{k',\, r\in R_2} \hat{T}^r_{(kk')} z^r_{(kk')}\geqslant 0 \quad (k\in K). \tag{4.13}$$

In any areal the local labour involved in production must not exceed the numbers available:

$$x_k\leqslant T^*_k \quad (k\in K). \tag{4.14}$$

Production facilities and the building industry must be provided with land, in whatever areal they are located:

$$u^{\text{ind}}_k-\sum_j s_j x_{jk}-s\sum_\nu z_{\nu k}\geqslant 0 \quad (k\in K). \tag{4.15}$$

The demand for land for civil construction must be met in each areal:

$$u^{\text{civ}}_k-\overline{S}\gamma\,(x_k+\overline{x}_k)\geqslant 0 \quad (k\in K). \tag{4.16}$$

The amount of land employed in each areal for industrial and civil construction must not exceed the amount available:

$$u^{\text{ind}}_k+u^{\text{civ}}_k\leqslant S_k\,(k\in K). \tag{4.17}$$

Industry must be provided with water in whatever areal it is located:

$$u_k-\sum_j W_j x_{jk}\leqslant 0 \quad (k\in K). \tag{4.18}$$

The condition of providing water for the population must be satisfied in each areal:

$$\overline{u}_k-W\gamma\,(x_k+\overline{x}_k)\geqslant 0 \quad (k\in K). \tag{4.19}$$

The choice of variant for consumption of water is determined in each areal in accordance with the demand of industry and of the population:

$$\sum_{r\in R_3} W^r_k u^r_k-u_k-\overline{u}_k\geqslant 0 \quad (k\in K). \tag{4.20}$$

For each areal there can be only one variant for its water supplies:

$$\sum_{r\in R_3} u_k^r \leqslant 1 \quad \text{with} \quad u_k^r = \begin{cases} 0 \\ 1 \end{cases} \quad (k\in K,\ r\in R_3). \tag{4.21}$$

The region's power grid must observe the annual load schedules for power both peak and base loads:

$$\sum_{m\in Q}\sum_{\varphi\in\Phi} V_{m\varphi t} - \sum_{j\in J}\sum_{k\in K} f_{jt}x_{jk} - \\ - \sum_{(kk')}\sum_{r\in R_2} f^r_{(kk')t} z^r_{(kk')} \geqslant N_t \quad (t=1,\ 2). \tag{4.22}$$

The size of the bases' load generated in the region's grid must be determined in accordance with the demand of both the optimised and the unconsidered facilities of industry and the infrastructure:

$$\sum_{m\in Q}\sum_{\varphi\in\Phi} h_t V_{m\varphi t} - \sum_{j\in J}\sum_{k\in K} Э_{j2}x_{jk} - \\ - \sum_{(kk')}\sum_{r\in R_2} Э^r_{(kk')2} z^r_{(kk')} \geqslant E \quad (\text{with } t=2). \tag{4.23}$$

The capacities of the consuming and generating facilities of each power system in the region's grid must correspond, with due account of the system's links with the other systems of the grid:

$$\sum_{m\in Q}\sum_{t} v_{m\varphi t} - \sum_{j\in J}\sum_{k\in \overline{K}_\varphi}\sum_{t} f_{jt}x_{jk} - \sum_{kk'}\sum_{r\in R_2}\sum_{t} f^r_{(kk')t} z^r_{kk'} - \\ - \bar{f}\sum_{\nu}\sum_{k\in\overline{K}_\varphi} z_{\nu k} - f\gamma \sum_{k\in\overline{K}_\varphi} (x_k + \bar{x}_k) + \\ + \sum_{\varphi'} \overline{V}_{\varphi'\varphi} - \sum_{\varphi'} \bar{v}_{\varphi\varphi'} V_{\varphi\varphi'} \geqslant \widetilde{N}_\varphi \quad (\varphi\in\Phi). \tag{4.24}$$

The total amount of electricity generated for each power system must be determined in accordance with the demand of both the optimised and the unconsidered facilities of industry and the infrastructure, the system's links with other systems taking into account:

$$\sum_{m\in Q}\sum_{t} h_t v_{m\varphi t}-\sum_{j\in J}\sum_{k\in\overline{K}_\varphi}\text{Э}_j x_{jk}-\sum_{(kk')}\sum_{r\in R_2}\text{Э}^r_{(kk')}z^r_{kk'}-$$
$$-\overline{\text{Э}}\sum_{\nu}\sum_{k\in\overline{K}_\varphi} z_{\nu k}-\Pi\gamma\sum_{k\in\overline{K}_\varphi}(x_k+\overline{x}_k)+$$
$$+\sum_{\varphi'}\overline{u}_{\varphi'\varphi}-\sum_{\varphi'}\overline{v}_{\varphi\varphi'}u_{\varphi\varphi'}\geqslant E_\varphi \quad (\varphi\in\Phi). \qquad (4.25)$$

The capacity of each high voltage transmission line must be adequate for the possible extra supplies of current along it:

$$V_{\varphi\varphi'}+\overline{V}_{\varphi'\varphi}-U_{\varphi\varphi'}\leqslant\overline{N}_{(ss')} \quad \text{(for each pair } ss'\text{)}. \qquad (4.26)$$

The current-carrying capacity of transmission lines must be sufficient to ensure possible transmissions of extra current along them:

$$u_{\varphi\varphi'}+\overline{u}_{\varphi'\varphi}-hU_{\varphi\varphi'}\leqslant\overline{M}_{(\varphi\varphi')} \quad \text{(for each pair } \varphi\varphi'\text{)}. \qquad (4.27)$$

The total capacity of each hydroelectric station must not exceed its maximum possible capacity:

$$\sum_{t} V_{m\varphi t}\leqslant N_{m\varphi} \quad \text{(for all } \varphi \text{ with } m\in Q_1\text{)}. \qquad (4.28)$$

The maximum total of electricity generated by each hydroelectric station must not exceed its maximum possible generation:

$$\sum_{t} h_t V_{m\varphi t}\leqslant M_{m\varphi} \quad \text{(for all } \varphi \text{ with } m\in Q_1\text{)}. \qquad (4.29)$$

For each new hydroelectric station the established ratio of peak and base load generation must be observed:

$$V_{m\varphi 1}-h_m V_{m\varphi 2}\leqslant 0 \quad \text{(for all } \varphi \text{ with } m\in Q_1\text{)}. \qquad (4.30)$$

The aggregate inputs on building and operating all production activities considered, on the transport of output within the region and out of it, the development of sections of the interareal trunk transport system, the building and operation of power stations, the building of transmission lines, and the creation and functioning of the building industry, on providing services for the local and incoming population, on developing the area under industrial and civil construction, and on providing industry and the public with water must be minimised:

$$\begin{aligned}
&\sum_{j\in J}\sum_{k\in K} c_{jk}x_{jk} + \sum_{i\in I_2\cup I_3}\sum_{k\in K_1} \bar{c}_{ik}\bar{x}_{ik} + \\
&+\sum_{i\in I}\sum_{kk'} c_{ikk'}x_{ikk'} + \sum_{i\in J}\sum_{k'k} \bar{c}_{ik'k}\bar{x}_{ik'k} + \sum_{\nu\in Y}\sum_{k\in\widetilde{K}_\nu} \tilde{c}_{\nu k}z_{\nu k} + \\
&+\sum_{r\in R_1}\sum_{\nu\in Y} \hat{c}^{r}_{\nu}\bar{z}^{r}_{\nu} + \sum_{k\in K} c_k x_k + \sum_{k\in K} \bar{c}_k\bar{x}_k + \sum_{r\in R_2}\sum_{kk'} \hat{c}^{r}_{(kk')}z^{r}_{kk'} + \\
&+\sum_{t}\sum_{m\in Q}\sum_{\varphi\in\Phi} \tilde{c}_{m\varphi t}V_{m\varphi t} + c\Big(\sum_{\varphi\varphi'} V_{\varphi\varphi'} + \sum_{\varphi'\varphi}\overline{V}_{\varphi'\varphi}\Big) + \\
&+\bar{c}\Big(\sum_{\varphi\varphi'} u_{\varphi\varphi'} + \sum_{\varphi'\varphi}\bar{u}_{\varphi'\varphi}\Big) + \sum_{\varphi\varphi'} c^{\mathrm{new}}_{\varphi\varphi'}U_{\varphi\varphi'} + \sum_{k\in K} c^{\mathrm{ind}}_{k}u^{\mathrm{ind}}_{k} + \\
&+\sum_{k\in K} c^{\mathrm{civ}}_{k}u^{\mathrm{civ}}_{k} + \sum_{k\in K} c^{\mathrm{w}}_{k}u_k + \sum_{k\in K} \bar{c}^{\mathrm{w}}_{k}\bar{u}_k + \sum_{r\in R_3}\sum_{k\in K} c^{r\mathrm{w}}_{k}u^{r}_{k} = \min.
\end{aligned} \tag{4.31}$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

The model proposed makes it possible to allow for the different character of the functioning and reciprocal connections of the facilities of the specialisation and complementary industries and elements of the inter-regional and general regional production infrastructure. The main conditions determining the choice of optimum variant of the solution in the computations are constraints (4.1) to (4.4) inclusive, which formalise the requirement of meeting the output targets of the specialisation industries (the main aim of the region) and the technological links taken into account in the problem of production activities.

The indices of inter-regional connections (export of finished products from the region; assignments for the routes and import of raw materials and the output of complementary industries) have a substantial effect on the location of new plants in the region, which is reflected in the model by constraints (4.1), (4.3), and (4.5) for border areals. The effect of inter-regional links is taken into account for interior areals through the variables for interareal exchange of output and raw materials $x_{ikk'}$ and $\overline{x}_{ik'k}$ [constraints (4.2) and (4.4)].

The distinguishing between border and interior areals is a specific feature of the model at this level. It is necessary for a more correct choice of variant for the functioning of the trunk transport system in accordance with the location plan for new production activities.

In addition, constraint (4.5) makes it possible to bring out the effectiveness of building complementary industries in the region and utilising local raw material resources.

Location models, as a rule, include constraints on the capacity of the production facilities. In contrast to them, the model proposed here includes such limitations only for facilities concerned with extracting raw materials, whose development depends on the technological possibilities [constraint (4.6)]. It is not expedient to limit the concentration of the capacities of the production activities of the specialisation and complementary industries, because the choice of areal for their location is solved in the problem, and not the choice of specific sub-areal (and given scale).

The formulation of the problem for defining the spatial structure of an economic region's economy also presupposes optimisation of industrial-transport connections. The density and configuration of the transport system, its technical equipping, and the volume and directions of the freight traffic through the region are taken into account in the model. Transport is represented in two aspects: namely, (a) by variants of the carrying capacity of each section $\Pi^{r}_{(kk')}$, and (b) by the possible freights of output of the optimised production activities on each section. It should be noted that the inclusion of independent variables for the interareal exchange of output makes it possible to avoid giving the modes of operation of plants that differ in their connections with suppliers of raw materials and other supplies. In addition, the transport routes for delivery of raw materials and output to the areal of consumption are not fixed before the problem is solved. This is not only expedient in the case in which we have to deal with a complex configuration of the region's transport system. Such a formalisation allows us to obtain fuller information for analysing the scheme built up in the optimum plan for providing industry with raw materials and other supplies.

The effect of the transport factor on the choice of scheme for the location of industry and the shaping of the transport system in accordance with the location of production facilities within the region are taken into account in the model through the system constraints (4.9) to (4.12) inclusive.

Since the volume of transit freight is given, the choice of variant of the carrying capacity of a section will mainly be determined by the variables $x_{ikk'}$ and $\bar{x}_{ik'k}$, which are included in the optimum plan [constraints (4.10) and (4.11)].

The inequality (4.12) reflects the technical conditions of the functioning of a transport section by variants of its development.

The provision of labour for new production activities in an areal, and the inputs connected with attracting labour and providing services for it, have a big effect on the location of these activities. These factors are reflected in the model through constraints (4.13) and (4.14). Constraint (4.13) allows us to compare areals by the inputs connected with the functioning of plants as regards supplying them with labour. The left-hand side of the constraint formulates the areal's demand for labour in relation to the production activities located in it, the volume of transport work, and the volume of building. The demand for labour in the service industries and in the farming areas around towns is not explicitly reflected in the model. It is taken into account through the inputs on attracting labour and providing services for the population.

Analysis of the function designators with the variables x_k and $\bar{x}_k$ indicates that the areal's demand will be primarily satisfied, in accordance with the criterion adopted, from local reserves. This means that constraints (4.13) and (4.14) could be amalgamated by excluding (4.14) and substituting T_k^* for the right-hand side of constraint (4.13). But the inclusion of independent variables for extra labour pools in the model is due to the need to allow for differentiation of the inputs connected with drawing local manpower into production in the various areals of the region.

The effectiveness of the location of new production activities by areals of the region is substantially determined by the availability of areas suitable for industrial and civil building, and also by the inputs on preparing these areas in accordance with the purpose for which they will be used, for which it is necessary to evaluate the withdrawal of land suitable for farming. In that case farming costs may be raised in the areal through a shift to poorer land, involving a reduction of inputs on building industrial facilities or vice versa. A similar situation arises when land is allocated for industrial or civil building. These factors are taken into account in the model through the system of constraints (4.15) to (4.17).

Water resources are reflected in the model by the consumption both of industry and the public and by the variants for supplying the areal. The provision of water for an areal is taken into account before the problem for the formulation of variants of water supply is resolved.

The principle of including water resources in the model [constraints (4.18) to (4.21)] is the same as for land. The allocation of water between industries and the population is taken into account through inequalities (4.18) and (4.19). The introduction of two types of constraint into the model is due to the differentiation of inputs on water supply according to the purpose for which the water is to be used.

When there is an increase in total consumption of water in an areal $(u_k + \bar{u}_k)$, inequality (4.20) makes it possible to decide whether to choose the next variant of water supply with an increase in outlays or to shift such-and-such a production activity to other areals. In this way inequalities (4.18) to (4.20) reflect both the competition of water consumers within an areal and the competition of areals according to the conditions of supply.

The choice of variant of the scheme for locating production activities is greatly affected by the inputs on building work carried out in different areals by different building organisations. The building industry is considered in the model in two aspects: namely, by the capacity of the firms and by the demands on their capacity by areals of the region.

The capacities of the building industry needed to meet the demand of industry and of the population for building work are determined in each areal through constraint (4.7), in which the competition between areals for use of the building industry's capacities (z_{vk}) is allowed for, as well as the competition between consumers in an areal (x_{jk}, x_k, and $\overline{x}_k$). In that connection variants for development of the capacity of each building depot are provided for in the model (constraints 4.8 and 4.9).

The power factor has a big effect on the location of industry in a region. The interconnections of the optimised production activities and the generating facilities of the regional grid are provided for in the model by the system of inequalities (4.22) to (4.25). Their inclusion is necessitated by the need not only to allow for the effect of the power factor on the location of new plants (especially of power-intensive production activities), but also to make a correct choice of variant for functioning of the power grid in accordance with the volume and pattern of power consumption in the system. The introduction of two types of balance (by capacity and by electricity) is associated with the need to determine the operating conditions of power stations from the standpoint both of the individual power systems and of the regional grid. Constraints (4.22) and (4.23) are used to allow for the direct and feedback connections between the functioning of the grid and the location of new production activities within the region.

The model considers possible variants of operation for each power station, differing according to the annual number of hours its capacity is utilised. The formulation of the problem presupposes that any combination of given variants for each station can be included in the optimum plan. The operation regimes of power stations are thus determined indirectly, in the solution of the problem, according to the consumers envisaged. Constraints of the type of (4.30) are included in the model for new hydroelectric stations; these ensure meeting the ratio between the generation of peak and base loads, which is needed so as to formulate the full inputs on new stations correctly in the objective function of the linear model.

The functioning of the whole aggregate of generating facilities as a single power system is ensured by the existence of a transmission grid, which is reflected in the model by the system of inequalities (4.26) and (4.27). This makes it possible to allow for the reciprocal effect of the location of the whole aggregate of generating facilities and consumers.

The extent of the provision of each power system with its own power is checked by means of constraints (4.24) and (4.25) in the case of new production activities being sited in its area; the size of the shortfall of power in the system and the means of covering it are also determined by these constraints.

Thus the effect of the main factors on shaping the scheme of the region's spatial structure is taken into account directly through the system of inequalities (4.1) to (4.30) inclusive, and through the indices of the criterion of optimality (4.31).

The model adduced here was employed to solve problems for Krasnoyarsk Territory and Irkutsk Region. The solutions pursued practical ends and were provided with sufficiently detailed data enabling the model to be employed without alterations or assumptions. The results of the computations indicated the usefulness of employing this model to optimise the spatial structure of a system of territorial industrial complexes.

5

MODEL FOR FORECASTING THE MAIN PARAMETERS OF A COMPLEX

N. I. Larina

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

The problem of optimising the main parameters of an individual territorial industrial complex consists in *determining the balanced variant of the development of the facilities of the specialisation and complementary industries and production and social infrastructure that will require the minimum input of scarce national resources.* In this problem attention is mainly concentrated on allowing for the time factor and proportionality in the development of the interconnected elements of the complex's economy.

To solve the problem we propose using a pointwise linear model describing the intracomplex conditions of production and distribution of output (resources) in each of the intervals of the plan period considered. The model's structure depends on the hierarchy adopted of the economic problems of long-term territorial-industrial planning, the sequence taken for their solution, and the types of model used.

As has already been indicated in Part I, the composition of the specialisation industries of each of a region's complexes, and their scale of development at the end of the plan period, are found from computations made with the model of the spatial structure of the economic region's system of complexes. In the following stage variants are selected for building enterprises of the specialisation industries that will be effective from the standpoint of the complex. At the same time the total demand for capital investment and manpower, and their allocation among the elements of the economy in different periods, are made more precise for the whole plan period.

The main objects of the investigation are the specialisation industries, complementary production activities, the production and social infrastructure, manpower, and natural resources.

The specialisation industries are represented by separate enterprises (existing, building, and projected), their scale of development at the end of the plan period being given. Variants of their development are differentiated by duration, the time of the beginning and completion of building, and the rate of building up their capacity.

In order to analyse all possible trends of employing scarce economic resources, the model considers variants of the development of the specialisation and complementary industries and of the infrastructure, which are also differentiated by indices of their labour and capital intensiveness. The variants are formulated so that choice of any one of them, or of a linear combination of several, will ensure fulfilment of the production targets for output of the specialisation industries.

The scale of development of complementary industries and of the infrastructure depends on the intracomplex demand for their products (services) and is determined in the course of the solution. If demand for their output has not declined by the end of the plan period, the growth of capacity can be indicated through variants of the growth of output by periods. This form of notation, however, will not work for transport and the building industry. The conditions of the development of transport cannot be reflected in a pointwise model. The development conditions of the building industry, on the contrary, must be indicated in more detail, since the dynamics of its formation and use of its capacity depends on various factors.

Under the formulation of the problem adopted the building of enterprises of the specialisation industries must be completed by the end of the plan period; the intracomplex demand for building work will consequently decrease after completion of their construction. In that case the dynamics of the use of the building industry's capacity can be reflected through the volume of building work in the separate periods, while the dynamics of the formation of its capacity can be represented through the dynamics of the capital investment needed to increase fixed productive assets (to calculate which the reserves and life of previously built capacities are employed).

There is no need for a different way of fixing the conditions of development of the complementary industries and infrastructure (except the building industry) in the model, because the degree of aggregation of the indices of output (services) and exploitation of resources for them is the same at this stage of the investigation. It follows from the target distribution of output and services within a complex, however, that a group of complementary industries and sectors of the infrastructure must be singled out, whose scale of development depends on the size of the population and the indices of living standards adopted. The rate at which the capacities of this group of industries and sectors are created is indicated separately.

The size of the population is determined by periods, taking the influx of labour into account. The possible influx of labour, however, is limited and must not exceed the values obtained from the model of the spatial structure of the system of complexes. By supposing that arriving specialists remain in the complex, we can indicate the rate of demand for imported labour through the indices of the numerical growth of manpower by periods. The possible exodus of population is not considered in the model, and the numerical size of manpower reserves is determined by separate periods. The optimality criterion is the requirement of minimising the current and capital costs needed to meet the targets for building enterprises of the specialisation industries, given proportional development of all elements of the complex's economy.

The rate of building enterprises of the specialisation and complementary industries and infrastructure and the rate and scale of exploitation of scarce resources depend on the previously established structure of the complex's economy and the scale of possible exploitation of local natural resources. These factors are taken into account when the vector of the model's constraints is being formulated.

The following notation is employed in the mathematical writing of the model.

INDICES AND SETS

i, j —indices of the enterprises (plants) of the specialisation industries ($i, j \in I_1$);

I_1 —set of enterprises considered;

p, ν —indices of the complementary production activities and of the infrastructure ($p \in I_2$, $\nu \in I_3$);

I_2 —the group of complementary industries and sectors of the infrastructure serving industry and the public;

I_3 —the group of complementary industries and sectors of the infrastructure serving the public;

r —variant of the development of an enterprise (plant, sector) ($r \in R_j$, $r \in R_p$, $r \in R_\nu$);

R_j —set of development variants for enterprise j;

R_p —set of development variants for production activity p;

R_ν —set of development variants for production activity ν;

t, φ —indices of the intervals of the plan period reviewed ($\varphi = 1, \ldots, T$; $t = 1, \ldots, T$);

λ_{pr} —building time-lag in production activity p for variant r of its development;

$\lambda_{\nu r}$ —building time-lag in production activity ν for variant r of its development;

λ —time-lag in building.

TECHNOLOGICAL COEFFICIENTS OF THE MATRIX

A_{ii}^{tr} —volume of output of product i by enterprise i in period t with development variant r;

A_{ij}^{tr} —volume of consumption of product i by enterprise j in period t with development variant r;

A_{pj}^{tr} —volume of consumption of product p by enterprise j in period t with development variant r;

F_j^{tr} —volume of building work on enterprise j in period t with development variant r;

L_j^{tr} —enterprise j's demand for labour in period t with development variant r;

a_{ip}^{tr} —unit input of product i in the output of product (service) p in period t with production development variant r;

$a_{pp'}^{tr'}$ —unit input of product p in the output of product (service) p' in period t with production development variant r';

f_p^{tr} —volume of building work in the unit capital costs of period t allocated to ensure growth of output in period $(t + \lambda_{pr})$ with development variant r of production activity p;

l_p^{tr} —unit input of labour in period t under development variant r of production activity p;

$a_{i\nu}^{tr}$ —unit input of product i for output of product (service) ν in period t under development variant r of production activity ν.

$a_{p\nu}^{tr}$ —unit input of product (service) p in the output of product (service) ν in period t under development variant r of production activity ν;

f_ν^{tr} —volume of building work in the unit capital costs of period t allocated to ensure growth of output in period $(t + \lambda_{\nu r})$ under development variant r of production activity ν;

l_ν^{tr} —unit input of labour in period t under development variant r of production activity ν;

μ_i^t —unit input of product i in period t for building in the complex;

μ_p^t —unit input of product (service) p in period t for building in the complex;

μ^t —proportion of building work to develop the building industry in the capital investments of period t;

$d^{\varphi t}$ —index of the building work in period t ensured by assets of the building industry on which capital investments were made in period φ;

l^t —unit labour intensity of building work in period t;

γ_i^t —norm for consumption of product i in time t per 1000 inhabitants of the complex;

γ_ν^t —norm for consumption of product (service) ν in period t per 1000 inhabitants of the complex;

γ_p^t —consumption norm of product (service) p in period t per 1000 inhabitants of the complex;

α^t —forecast (estimated) average size of families in period t;

$\varkappa^{t\varphi}$ —rate of growth of the labour force in period t compared with period φ, taking natural growth of the population into account.

CONSTRAINTS

b_i^t —demand of the complex's existing industries for product i in period t;

b_p^t —demand of existing industries for product (service) p in period t;

b_ν^t —demand of existing industries for product (service) ν in period t;

F^t —demand of existing industries for building work in period t;

Φ^t —volume of building work in period t carried out with assets of the building industry made available in the pre-plan period;

L^t —local labour reserves in period t;

H^t —size of the local population in period t;

V —size of the permissible influx of labour for the whole plan period;

D_p^t —maximum possible output of production activity p in period t.

FUNCTION DESIGNATORS

U_j^r —total value of discounted costs of building and operating enterprise j (ignoring inputs on the products and services of comlpementary industries and sectors of the infrastructure), under variant r;

$c_{pr}^{t\varphi}$ —discounted unit current inputs on output of goods (services) of industrial activity p in period t under variant r of the scale of growth of production capacity in period φ;

$q^{\varphi-\lambda_{pr},\,\varphi}$ —discounted unit capital investments of period $(\varphi - \lambda_{pr})$ to ensure growth of output (services) of production activity p in period φ under variant r;

$c_{\nu r}^{t\varphi}$ —discounted unit current costs of output of goods (services) of production activity ν in period t under variant r within the scale of growth of production capacity in period φ;

$q^{\varphi-\lambda_{\nu r},\,\varphi}$ —discounted unit capital investments in period $(\varphi - \lambda_{\nu r})$ to ensure growth of output of production activity ν under variant r in period φ;

σ^{φ} —discount rate of capital investments of period φ;

s^{φ} —discounted capital investments to provide services for 1000 persons in period φ.

UNKNOWNS

x_i^r, x_j^r —rate of development i of enterprise j of a specialisation industry by variant r;

y_{pr}^{φ} —growth of output (services) in period φ under variant r for the development of production activity p;

$y^{\varphi}_{\nu r}$ —growth of output (services) in period φ under variant r for the development of production activity ν;
y^{t} —volume of building work in the complex in period t;
k^{φ} —capital investment in period φ on developing the building industry;
h^{t} —population of the complex in period t;
v^{φ} —demand for incoming labour in period φ;
w^{t} —labour reserves not employed in period t.

The model is written mathematically in the notation adopted, as follows:

Find $x^{r}_{i} \geqslant 0$, $x^{r}_{j} \geqslant 0$, $y^{\varphi}_{pr} \geqslant 0$, $y^{\varphi}_{\nu r} \geqslant 0$, $y^{t} \geqslant 0$, $k^{\varphi} \geqslant 0$, $h^{t} \geqslant 0$, $v^{\varphi} \geqslant 0$, $w^{t} \geqslant 0$, such as satisfy the constraints given below.

For each of the periods considered the following input-output matrices of the production and distribution of output (services) are complied with:

(a) for specialisation industries

$$\sum_{r\in R_i} A^{tr}_{ii}x^{r}_{i} - \sum_{I\in J_{2\setminus i}} \sum_{r\in R_j} A^{tr}_{ij}x^{r}_{j} - \sum_{p\in I_2} \sum_{r\in R_p} a^{tr}_{ip} \sum_{\varphi=1}^{t} y^{\varphi}_{pr} -$$

$$- \sum_{\nu\in I_3} \sum_{r\in R_\nu} a^{tr}_{i\nu} \sum_{\varphi=1}^{t} y^{\varphi}_{\nu r} - \mu^{t}_{i}y^{t} - \gamma^{t}_{i}h^{t} \geqslant b^{t}_{i}$$

$$(i\in I_1,\ t=1,\ \ldots,\ T); \tag{5.1}$$

(b) for complementary industries and the infrastructure

$$\sum_{r\in R_p} \sum_{\varphi=1}^{t} y^{\varphi}_{pr} - \sum_{j\in I_1} \sum_{r\in R_j} A^{tr}_{pj}x^{r}_{j} -$$

$$- \sum_{p'\in I_{2\setminus p}} \sum_{r'\in R_{p'}} a^{tr'}_{pp'} \sum_{\varphi=1}^{t} y^{\varphi}_{p'r'} - \sum_{\nu\in I_3} \sum_{r\in R_\nu} a^{tr}_{p\nu} \sum_{\varphi=1} y^{\varphi}_{\nu r} -$$

$$- \gamma^{t}_{p}h^{t} - \mu^{t}_{p}y^{t} = b^{t}_{p} \quad (p\in I_2,\ t=1,\ \ldots,\ T); \tag{5.2}$$

$$\sum_{r\in R_\nu} \sum_{\varphi=1}^{t} y^{\varphi}_{\nu r} - \gamma^{t}_{\nu}h^{t} = b^{t}_{\nu} \quad (\nu\in I_3,\ t=1,\ \ldots,\ T). \tag{5.3}$$

For each of the periods considered the complex's total demand for building work is determined:

$$y^t - \sum_{j\in I_1}\sum_{r\in R_j} F_j^{tr} x_j^r - \sum_{p\in I_2}\sum_{r\in R_p} f_p^{tr} y_{pr}^{(t+\lambda_{pr})} -$$

$$- \sum_{\nu\in I_3}\sum_{r\in R_\nu} f_\nu^{tr} y^{(t+\lambda_{\nu r})} - \mu^t k^{(t+\lambda)} \geqslant F^t \quad (t=1, \ldots, T). \qquad (5.4)$$

Development of the building industry depends on the possible volume of building work by periods:

$$\sum_{\varphi\in 1}^{t} d^{\varphi t} k^{\varphi} - y^t \geqslant -\Phi^t \quad (t=1, \ldots, T). \qquad (5.5)$$

The following manpower balances are complied with for each of the periods considered:

$$\sum_{j\in I_1}\sum_{r\in R_j} L_j^{tr} x_j^r + \sum_{p\in I_2}\sum_{r\in R_p} l_p^{tr} \sum_{\varphi=1}^{t} y_{pr}^{\varphi} +$$

$$+ \sum_{\nu\in I_3}\sum_{r\in R_\nu} l_\nu^{tr} \sum_{\varphi=1}^{t} y_{\nu r}^{\varphi} + l^t \sum_{\varphi=1}^{t} d^{\varphi t} k^{\varphi} -$$

$$- \sum_{\varphi=1}^{t} \varkappa^{t\varphi} v^{\varphi} + w^t = L^t \quad (t=1, \ldots, T). \qquad (5.6)$$

The complex's population in each period is established, allowing for the influx of labour:

$$h^t - \sum_{\varphi=1}^{t} \alpha^t v^{\varphi} = H^t \quad (t=1, \ldots, T). \qquad (5.7)$$

The possible influx of labour in all periods is limited:

$$\sum_{\varphi=1}^{T} v^{\varphi} \leqslant V. \qquad (5.8)$$

The scale of development of the complementary industries processing intracomplex raw material resources is limited by the latter's commercial reserves:

$$\sum_{r\in R_p}\sum_{\varphi=1}^{t} y_{pr}^{\varphi}\leqslant D_p^t \quad (p\in I_2,\ t=1,\ \ldots,\ T). \tag{5.9}$$

The targets for the building of enterprises of the specialisation industries in the complex must be met:

$$\sum_{r\in R_j} x_j^r\geqslant 1 \quad (j\in I_1). \tag{5.10}$$

For this purpose the sum total of discounted current and capital inputs on building and operating the given set of enterprises of the specialisation industries in the plan period, and on developing the complementary industries and infrastructure to the required scale must be minimal:

$$\sum_{j\in I_1}\sum_{r\in R_j} U_j^r x_j^r+\sum_{p\in I_2}\sum_{r\in R_p}\sum_{t=1}^{T}\sum_{\varphi=1}^{T}(c_{pr}^{t\varphi}+q^{\varphi-\lambda_{pr},\,\varphi})\,y_{pr}^{\varphi}+$$
$$+\sum_{\varphi=1}^{T} s^{\varphi}v^{\varphi}+\sum_{\nu\in I_3}\sum_{r\in R_\nu}\sum_{t=1}^{T}\sum_{\varphi=1}^{T}(c_{\nu r}^{t\varphi}+q^{\varphi-\lambda_{\nu r},\,\varphi})\,y_{\nu r}^{\varphi}+$$
$$+\sum_{\varphi=1}^{T}\sigma^{\varphi}k^{\varphi}\rightarrow \min. \tag{5.11}$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

So that proportions will be observed in the development of interconnected production activities when variants are being selected for building enterprises of the specialisation industries and determining the scale of development of the complementary industries and infrastructure, input-output matrices of the production and distribution of their output in each of the intervals of the plan period are considered in the model [constraints (5.1) to (5.3) inclusive].

The volume of production in each interval is established in accordance with the demand for the product. The possible export from the region of output of specialisation industry enterprises in the separate intervals is determined as the difference between the volumes of production and intracomplex demand. If the information on the national demand for output of the specialisation industries in the separate intervals of the plan period is available before the problem is solved, this information is introduced through index b_i^t as a target for shipping output.

The systems of equations (5.2) and (5.3) make it possible to find the intracomplex demand for output of the complementary industries and services of the infrastructure and to determine the necessary increment of output by periods. The scale of development of these industries at the beginning of the plan period is taken into account through calculating indices b_p^t and b_v^t. Information on the necessary export or possible import of output of the complementary industries by intervals is introduced by means of these indices. The demand of the complex's previously created production activities for services of the building industry is taken into account through index F^t if it is not known before solution of the problem. This index mainly embraces the demand of enterprises under construction or being reconstructed. When the volume of capital investments (k^φ) on developing the building industry is being determined attention is paid to the reserve capacity Φ^t previously created. The sign $\geqslant$ in constraint (5.5) indicates that capacity can be released from the building industry as the volume of building work in the complex declines. The volume of reserve capacity in the separate intervals is not explicitly represented in the model since the directions of their use are not being studied in this problem. The scale of the demand for output (and services) in building depends on the volume of building work in the separate intervals (y^t). The demand for labour is determined through the index of existing assets, i.e. is not reduced as intracomplex demand for building work declines.

Labour input-output matrices for the intervals [constraint (5.6)] enable the intersectoral distribution of labour in the complex to be analysed and the demand for incoming labour found. When the index of local reserves of labour L^t is calculated, allowance is made for the demand of those of the complex's industries (production activities) whose development is not studied in the model (transport, operating enterprises and plants and those under construction, etc.). Because the amount of incoming labour for all periods is limited [constraint (5.8)], the effect of the limitation of manpower on the shaping of the complex's industrial structure is taken into account through the input-output matrix of the distribution of labour by elements of the complex's economy in the separate periods [constraint (5.6)].

The complex's population in the different intervals of the plan period is determined by taking account of the size of the local population, the influx of labour, and the coefficients adopted for average size of families and natural increase of population [constraint (5.7)]. The index of the complex's total population (h^t) is introduced so as to bring out the rate of change and scale of development of the social infrastructure and complementary industries serving the public [constraint (5.3)].

As indicated above, the complementary industries serving the specialisation industries are included in set I_2. The scale of development of some of them is determined not only by the size of the demand for their output [constraint (5.2)] but also depends on the permitted volumes of exploitation of local natural resources [see constraint (5.9)]. The upper limits for the development of the production activities (industries) of set I_3 can be shown in the same way.

The target for scale of development of the complex's specialisation industries is introduced through constraint (5.10), the sign of which indicates the possibility of output above the assigned volumes of these industries. Such a situation can arise through an imbalance in the production and distribution of output in certain periods if the possible rates of growth of capacity have not been taken fully into account when the variants were being formulated.

The optimality criterion (5.11) makes it possible to choose variants for building specialisation enterprises that call for minimum current and capital costs of the proportional development of all elements of the complex's economy. The sum total of capital investments corresponding to the optimum plan, and their allocation by industries and periods, also present interest.

If the complex calls for more investment than was allocated when the line of specialisation was decided on (structure and scale of development of the specialisation industries), it will be necessary to make a supplementary analysis of the dynamics of building up its economy, by means of computations on a model with a criterion of minimum capital investment. In that case the model's structure will be altered a little, and the links between investment, fixed productive assets, and the dynamics of output will be described in greater detail.

6

MODEL OF THE SPATIAL STRUCTURE OF A COMPLEX

V. S. Zverev and M. A. Malinovskaya

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

The links of territorial industrial complexes within an economic region and with facilities in other regions, their production specialisation, the scale of exploitation of their resources, and the location of facilities of the general regional infrastructure in them, are determined by means of a model of the spatial structure of a region's system of complexes and a model for forecasting a complex's main parameters.

These general parameters, however, cannot fully define the scheme of the spatial organisation of a complex's economy, since it is necessary, for that, to consider the conditions inside the complex in more detail (location of facilities by industrial centres; development and location of the intra-complex infrastructure).

At this stage, therefore, the problem is *to define the location scheme of enterprises and plants, the structure and production-economic connections of industrial centres, the location of the main elements of the infrastructure and of the population, the exploitation and development of the transport system, given minimisation of costs of the building and functioning of all the elements of the complex's economy concerned.*

When solving the main problem of this stage of the investigation it is necessary to allow for the effect of the following factors:

(1) the mutual location of enterprises over the complex;

(2) the production (technological) connections between enterprises and production activities;

(3) enterprises' possible use of different sources of resources (individual deposits of the complex, resources brought in from outside, and resources of the sub-areals figure as such);

(4) the need for migration of labour to the building sites of the enterprises and production activities;

(5) the allotted proportions between categories of labour in industrial centres;

(6) the carrying capacity of the separate sections of the transport system and the opportunities for increasing it under different variants of reconstruction.

The effect of these factors must be evaluated from the standpoint of the minimum aggregate costs of functioning of the whole complex, and must be reflected in the model directly, through the introduction of appropriate systems of equations and inequalities, or indirectly through the function designators of the objective function.

The objects investigated in the model of a complex's spatial structure are the enterprises of the specialisation industries (both existing ones and new ones), individual enterprises or groups of enterprises of the complementary industries, the areas of industrial centres suitable for industrial and civil building, the sections of the transport system, agriculture, and manpower.

The field of acceptable solution for each of these objects is determined by various conditions: namely, for enterprises of the specialisation industries, by the possibility of selecting sites and production links according to the exploitation of the complex's resources; for complementary industries, by the existence of, or possibility of, providing resources within the complex's industrial centres; for sectors of the area of a centre, by the level of development of its territory; for sections of the transport system, by the level of exploitation of existing carrying capacity and the opportunities for reconstructing sections; for agriculture, by the possibility of choosing its location (within industrial centres or in undeveloped areas of the complex); for manpower, by the scale and structure of the possible demand of the industrial centres for labour.

The proposed formulation of the problem is embodied in a mathematical economic model, for which the following notation is used.

INDICES AND SETS

i, i', i'' —indices of products or types of raw material ($i \in I_1 \cup I_2 \cup I_3$, $i' \in I_4 \cup I_3$, $i'' \in I_5$);

I_1 —set of the output of enterprises of the specialisation industries;

I_2 —set of the output of enterprises of the specialisation industries existing at the beginning of the plan period;

I_3 —set of products brought in from outside the complex;

I_4 —set of types of raw material produced by enterprises of complementary industries;

I_5 —set of the products produced by enterprises of the complementary industries whose siting and development are dictated by the demand of enterprises located in the same or a neighbouring area;

j, j', j'' —indices of enterprises or production activities ($j \in J_1 \cup J_2$);

J_1 —set of new enterprises of the specialisation industries;

J_2 —set of specialisation industry enterprises existing at the beginning of the plan period;

J_4 —set of enterprises producing product $j' \in I_4$;

J_5 —set of enterprises producing product $j'' \in I_5$;

k, k' —indices of sub-areals ($k, k' \in K \cup K_j \cup \cup K_p \cup K_{j''k}$);

K —set of the sub-areals of the complex concerned;

K_j —set of sub-areals of possible sites for enterprise j;

K_p —set of sub-areals whose enterprises consume the resources of deposit p;

$K_{j''k}$ —set of sub-areals whose enterprises consume product j'' of an enterprise located in sub-areal k;

r —variant of the functioning of production facilities or exploitation of resources ($r \in \in R_{jk} \cup R'$, none of the variants of the functioning of facilities of the specialisation industries being differentiated by capacity);

R_{jk} —set of variants of the functioning of enterprise j in sub-areal k;

R' —set of variants for reconstruction of sections of the transport system;

J^* —set of enterprises that cannot be sited in the same sub-areal for sanitary and hygienic reasons;

$J^p_{i'}$ —set of enterprises of the specialisation industries consuming raw material i' from deposit p;

J_k —set of specialisation industry enterprises that can be located in sub-areal k;

p —index of a deposit of raw materials or fuel $p \in P_{i'} \bigcup P_{i'k}$;

$P_{i'}$ —set of deposits of type i' raw material or fuel;

$P_{i'k}$ —set of deposits of type i' raw material or fuel located in sub-areal k;

(kk') —index of a section of the transport system $[(kk') \in (KK')]$;

(KK') —set of the sections into which the complex's whole transport system is broken down;

d —index of a route for shipping output out of the complex $[d \in D_i,\ D\ (kk')]$;

D_i —set of routes for shipping out product i;

$D_{(kk')}$ —set of routes for shipping output that pass over section (kk');

μ —index of a category of manpower;

Θ —set of categories of manpower;

n_1 —index of a group of sectors of the area $(n_1 \in N_1^k)$;

N_1^k —set of groups of sectors of sub-areal k;

n_2 —index of groups of water sources $(n_2 \in N_2^k)$;

N_2^k —set of groups of water sources in sub-areal k.

MATRIX COEFFICIENTS

$A^r_{ij},\ A^r_{i''j}\ (a^r_{ij})$—volume of production (consumption) of product i or i'' in enterprise j operating by variant r; $a_{ii'}$, $a_{ii''}$, $a_{i'i''}$, $a_{i''i'}$, $a_{i''_0 i'}$, $a_{i'_0 i''}$ are the coefficients of the direct inputs of i- or i''_0-type products or of i'-type raw materials on producing an i''-type product or i'-type raw material;

γ —demographic load factor;

ε_μ —proportion of workers of category μ employed in the service industries in the total population;

γ^* —coefficient characterising the ratio of male and female labour in the population;

$f_{i''}$, $f_{i'}$ —norm of the annual consumption of product i'' or raw material i' per head of population;

η —value of the fixed assets of facilities of the social infrastructure and public utilities, as envisaged by town-planning norms per head of population;

$A^r_{i'pj}$ —volume of the consumption of raw material i' of deposit p at enterprise j, functioning by variant r;

$A^{kr}_{j(kk')}$ —volume of freights of raw material or fuel on section (kk') of the complex's transport system needed to ensure operation of enterprise j (located in sub-areal k) by variant r;

$A^r_{j(kk')}$ —the same, for an existing enterprise;

$T^{r\mu}_j$ —number of workers of category μ at enterprise j by variant r;

$T^\mu_{i'p}$, $T^\mu_{i''}$ —number of workers of category μ of a complementary industry producing a unit of i'-type raw material at deposit p, or of an i''-type product;

$a_{i''(kk')}$ —volume of freight on section (kk') associated with the manufacture of a unit of product by a complementary industry i'';

S_j^1 —area of land needed to site enterprise j;

$\overline{S}^1$ —per capita area of the territory;

S_j^2 —volume of water needed by enterprise j;

$\overline{S}^2$ —volume of water needed per capita per annum.

CONSTRAINTS

$\Pi_{i'}$ —limit on bringing in raw material i' from outside the complex;

L_μ —total numbers of local manpower reserves of category μ not employed in the complex;

$\Pi^0_{(kk')}$ —reserve carrying capacity of existing sections (kk') of the complex's transport system;

$\Pi^r_{(kk')}$ —limiting annual carrying capacity of section (kk') of the transport system under reconstruction variant r $(r \in R')$;

$\Pi_{i'p}$ —limiting annual volume of extraction of i'-type raw material at deposit p;

$S_k^{1n_1}$ —area of free group n_1 land in sub-areal k suitable for industrial and civil construction, less land used for farming;

$S_k^{2n_2}$ —maximum possible volume of water from sources of group n_2 that can supply newly built facilities in sub-areal k;

F_k —value of the previously built fixed assets of facilities of the social amenities infrastructure in sub-areal k.

FUNCTION DESIGNATORS

c_j^{kr} —adjusted costs of the output of enterprise j located in sub-areal k and functioning by variant r;

$\bar{c}_1^{kn_1}$ —adjusted costs of developing group n_1 land earmarked for industrial construction in sub-areal k;

$c_{i'}$ —adjusted costs of bringing a unit of i'-type raw material from outside the complex;

$\bar{c}_2^{kn_1}$ —adjusted costs to develop group n_1 area earmarked for civil construction in sub-areal k;

$c_{i''k}$ —adjusted costs to produce a unit of product i'' in sub-areal k;

$c_{i'p}$ —adjusted costs per unit volume of extraction of i'-type raw material at deposit p;

$\bar{c}_k^1$ —adjusted costs of providing services for one person arriving in sub-areal k from outside the complex;

$\bar{c}_k^2$ —adjusted costs of providing services for one person coming to sub-areal k from another sub-areal within the complex;

$\tilde{c}_k$ —loss of manpower through non-employment in sub-areal k;

$c^0_{(kk')}$ —inputs to operate section (kk') of the complex's transport system, exploiting the section's reserve carrying capacity;

$c^r_{(kk')}$ —adjusted costs of providing additional carrying capacity on a section (kk') of the complex's transport system under reconstruction variant r;

$c_k^{n_2}$ —adjusted costs of water supplies for industry and the public in sub-areal k from group n_2 sources.

VARIABLES

x_j^{kr} —rate of operation of enterprise j in sub-areal k by mode r;

$y_{i'}$ —volume of i'-type raw material brought in from outside the complex;

$x_1^{kn_1}$ —area of n_1 land occupied by industrial building in sub-areal k;

$x_2^{kn_1}$ —area of group n_1 land occupied by civil building in sub-areal k;

$x^0_{(kk')}$ —exploitation of the reserve carrying capacity of section (kk') of the complex's transport system;

$x^r_{(kk')}$ —exploitation of additional carrying capacity of section (kk') of the complex's transport system under reconstruction variant r $(r \in R')$;

$\bar{x}_k^{n_2}$ —magnitude of the use of water resources from group-n_2 sources in sub-areal k;

$y_{i'p}$ —volume of i'-type raw material mined at deposit p;

$y^{jk}_{i''k'}$ —volume of output of product i'' in sub-areal k$'$ using output of enterprise j located in sub-areal k;

z^{jk}_{id} —export of i-type product of enterprise j from sub-areal k along route d;

z_{id} —export of i-type product along route d from existing enterprises;

$l_{k\mu}$ —numbers of local μ-category labour drawn to newly built production activities in sub-areal k, from the number served by the social amenities infrastructure;

$l^1_{k\mu}$ —numbers of μ-category labour brought into sub-areal k from outside the complex;

$l^2_{k\mu}$ —numbers of μ-category labour drawn to sub-areal k from other sub-areals of the complex;

$l_{k\mu_1}$ —quantity indicating how far the number of able-bodied men exceeds the given ratio in the composition of the employable population of sub-areal k;

$\bar{l}_{k\mu_2}$ —quantity indicating how far the number of able-bodied women exceeds the given ratio in the composition of the employable population of sub-areal k.

The mathematical economic model of the problem of a complex's spatial structure is written as follows, using the notation adopted:

Find the non-negative values of x_j^{kr}, $x_1^{kn_1}$, $x_2^{kn_1}$, $\bar{x}_k^{n_2}$, $y_{i'}$, $y_{i'p}$, $y_{i''k}^{ik'}$, $x_{(kk')}^{0}$, $x_{(kk')}^{r}$, z_{id}^{jk}, z_{id}, $l_{k\mu}$, $l_{k\mu}^{1}$, $l_{k\mu}^{2}$, $\bar{l}_{k\mu_1}$, $\bar{l}_{k\mu_2}$, such as will satisfy the following constraints.

Production and distribution of output must be balanced in all sub-areals in which facilities of the specialisation industries may be located, and also of all the products produced by them:

$$\sum_{r\in R_{j'k}} A_{ij'}^{r} x_{j'}^{kr} - \sum_{j\in J_1\cup J_2} \sum_{k\in K_j} \sum_{r\in R_{jk}} a_{ij}^{r} x_j^{kr} -$$

$$- \sum_{i'\in J_4} \sum_{p\in P_{i'}} a_{ii'} y_{i'p} - \sum_{i''\in J_5} \sum_{k'\in K} a_{ii''} y_{i''k'}^{jk} - \sum_{d\in D_i} z_{id}^{jk} = 0$$

$$(j' \neq j,\ j\in J_1\cup J_2,\ k\in K_{j'},\ i\in I_1\cup I_2). \tag{6.1}$$

Each of the given facilities of the specialisation industries must be within the confines of the complex;

$$\sum_{k\in K_j} \sum_{r\in R_{jk}} x_j^{kr} = 1 \quad (j\in J_1\cup J_2). \tag{6.2}$$

The development of facilities extracting raw materials and fuel at individual deposits must satisfy the complex's demand for their output, allowing for imports from outside:

$$y_{i'p} - \sum_{j\in J_{i'}^{p}} \sum_{k\in K_j} \sum_{r\in R_{ik}} A_{i'pj}^{r} x_j^{kr} - \sum_{i''\in J_5} \sum_{k\in K} \sum_{j\in J_1} \sum_{k\in K_j} a_{i'i''} y_{i''k'}^{jk} +$$

$$+ y_{i'} - f_{i'} \sum_{k\in K} \sum_{\mu\in\Theta} \gamma\,(l_{k\mu} + l_{k\mu}^{1} + l_{k\mu}^{2}) \geqslant 0$$

$$(p\in P_{i'},\ i'\in I_3\cup I_4). \tag{6.3}$$

The development of facilities servicing specialisation industries, other production activities, and the public must satisfy the demand for their product in each sub-areal of the complex:

$$\sum_{j\in J_1}\sum_{k\in K_j} y^{jk}_{i''_0k'} - \sum_{i''\in J_5}\sum_{k'\in K_{i''_0k'}}\sum_{j\in J_1}\sum_{k\in K_j} a_{i''_0i''}y^{jk}_{i''k'} -$$

$$-\sum_{k\in K_{i''_0k'}}\sum_{j\in J_k}\sum_{r\in R_{jk}} a^r_{i''_0j}x^{kr}_j - \sum_{k\in K_{i''_0k'}}\sum_{i'\in J_4}\sum_{p\in P_{i'k}} a_{i''_0i}y_{i'p} -$$

$$-f^{i''}_0 \sum_{k\in K_{i''_0k'}}\sum_{\mu\in\Theta} \gamma\,(l_{k\mu}+l^1_{k\mu}+l^2_{k\mu}) = 0$$

$$(i''_0\in J_5,\ k'\in K). \tag{6.4}$$

The numbers of local male and female labour drawn to newly created production activities in all the sub-areals must not exceed their availability for the complex as a whole:

$$\sum_{k\in K} l_{k\mu} + \sum_{k\in K} l^2_{k\mu} \leqslant L_\mu \quad (\mu\in\Theta). \tag{6.5}$$

The total freight traffic on each section of the complex's transport system must not exceed the section's carrying capacity:

$$\sum_{j\in J_1}\sum_{k\in K_j}\sum_{r\in R_{jk}} A^{kr}_{j(kk')}x^{kr}_j + \sum_{i''\in J_5}\sum_{k'\in K}\sum_{j\in J_1}\sum_{k\in K_j} a_{i''(kk')}y^{jk}_{i''k'} +$$

$$+\sum_{d\in D_{(kk')}}\sum_{j\in J_1}\sum_{k\in K_j}\sum_{i\in I_1} z^{jk}_{id} + \sum_{d\in D_{(kk')}}\sum_{i\in I_2} z_{id} -$$

$$-x^0_{(kk')} - \sum_{r\in R'} x^r_{(kk')} \leqslant 0$$

$$((kk')\in(KK')). \tag{6.6}$$

The rate of exploitation of each section of the complex's transport system is limited by the established limiting carrying capacity (including that for each variant of reconstruction):

$$\Pi^0_{(kk')} \leqslant \Pi^0_{(kk')} \qquad ((kk')\in(KK')); \tag{6.7}$$

$$x^r_{(kk')} \leqslant \Pi^r_{(kk')}\ ((k'k)\in(KK'),\ r\in R'). \tag{6.8}$$

Imports of each type of raw material from outside the complex must not exceed the established limits:

$$y_{i'} \leqslant \Pi_{i'} \quad (i'\in I_3). \tag{6.9}$$

The volume of extraction of each type of raw material from the complex's deposits must not exceed the established limiting amounts:

$$y_{i'p} \leqslant \Pi_{i'p} \quad (i' \in I_4;\ p \in P_{i'k},\ k \in K). \tag{6.10}$$

The demand for male and female labour of all facilities located in each sub-areal must be met:

$$\sum_{j \in J_1} \sum_{r \in R_{jk'}} T_j^{r\mu} x_j^{k'r} + \sum_{i' \in J_4} \sum_{p \in P_{i'k'}} T_{ip}^{\mu} y_{i'p} +$$
$$+ \sum_{i'' \in J_5} \sum_{j \in J_1} \sum_{k \in K_j} T_{i''}^{\mu} y_{i''k'}^{jk'} - (1 - \gamma\varepsilon_\mu)(l_{k'\mu} + l_{k'\mu}^1 + l_{k'\mu}^2) \leqslant 0$$
$$(\mu \in \Theta,\ k \in K). \tag{6.11}$$

The utilisation of previously created fixed assets of the social infrastructure in each sub-areal must not exceed their availability:

$$\eta \sum_{\mu \in \Theta} \gamma l_{k\mu} \leqslant F_k \quad (k \in K). \tag{6.12}$$

The sex structure of employment in each sub-areal is estimated taking the forecast ratio of men and women into account:

$$l_{k\mu_1} + l_{k\mu_1}^1 + l_{k\mu_1}^2 - \gamma^* (l_{k\mu_2} + l_{k\mu_2}^1 + l_{k\mu_2}^2) -$$
$$- \bar{l}_{k\mu_1} + \bar{l}_{k\mu_2} = 0 \quad (k \in K). \tag{6.13}$$

The land allocated in each sub-areal for industrial building must not be less than that needed for all the industrial facilities located in that sub-areal:

$$\sum_{n_1 \in N_1^k} x_1^{kn_1} - \sum_{j \in J_k} \sum_{r \in R_{jk'}} S_j^1 x_j^{k'r} -$$
$$- \sum_{i'' \in J_5} \sum_{j \in J_1} \sum_{k \in K_j} S_{i''}^1 y_{i''k}^{jk} \geqslant 0 \quad (k \in K). \tag{6.14}$$

The same condition must be satisfied for the areas earmarked for civil building:

$$\sum_{n_1 \in N_1^k} x_2^{kn_1} - \sum_{\mu \in \Theta} \gamma \bar{S}^1 (l_{k\mu} + l_{k\mu}^1 + l_{k\mu}^2) \geqslant 0 \quad (k \in K). \tag{6.15}$$

The total area in each group used in each sub-areal must not exceed the latter's free area:

$$x_1^{kn_1} + x_2^{kn_1} \leqslant S_k^{1n_1} \quad (k \in K,\ n_1 \in N_1^k). \tag{6.16}$$

The water requirements of all production facilities and of the public in each sub-areal must be met:

$$\sum_{n_2 \in N_2^k} \bar{x}_k^{n_2} - \sum_{j \in J^k} \sum_{r \in R_{jk}} S_j^2 x_j^{kr} - \sum_{i'' \in J_5} \sum_{j \in J_1} \sum_{k' \in K} S_{i''}^2 y_{i''k}^{jk'} -$$
$$- \sum_{\mu \in \Theta} \gamma \bar{S}^2 (l_{k\mu} + l_{k\mu}^1 + l_{k\mu}^2) \geqslant 0 \quad (k \in K). \tag{6.17}$$

The rate of exploitation of each water source must not exceed its maximum capacity:

$$x_k^{n_2} \leqslant S_k^{2n_2} \quad (k \in K,\ n_2 \in N_2^k). \tag{6.18}$$

Sanitation and hygiene conditions must be complied with in each sub-areal:

$$\sum_{j \in J^*} \sum_{r \in R_{jk}} x_j^{kr} \leqslant 1 \quad (k \in K). \tag{6.19}$$

Minimum aggregate inputs on the building and functioning of the complex's newly built facilities is ensured by satisfying constraints (6.1) to (6.19):

$$\sum_{j \in J^1} \sum_{k \in K_j} \sum_{r \in R_{jk}} c_j^{kr} x_j^{kr} + \sum_{p \in P_{i'}} \sum_{i' \in J_4} c_{i'p} y_{i'p} +$$
$$+ \sum_{i'' \in J_5} \sum_{k \in K} \sum_{j \in J_1} \sum_{k' \in K_j} c_{i''k} y_{i''k}^{jk'} + \sum_{i' \in I_3} c_{i'} y_{i'} +$$
$$+ \sum_{k \in K} \sum_{n_1 \in N_1^k} (\bar{c}_1^{kn_1} x_1^{kn_1} + \bar{c}_2^{kn_1} x_2^{kn_1}) + \sum_{k \in K} \sum_{n_2 \in N_2^k} c_k^{n_2} \bar{x}_k^{n_2} +$$
$$+ \sum_{(kk') \in (KK')} [c_{(kk')}^0 x_{(kk')}^0 + \sum_{r \in R'} c_{(kk')}^r x_{(kk')}^r] +$$
$$+ \sum_{k \in K} \bar{c}_k^1 \sum_{\mu \in \Theta} \gamma l_{k\mu}^1 + \sum_{k \in K} \bar{c}_k^2 \sum_{\mu \in \Theta} \gamma l_{k\mu}^2 +$$
$$+ \sum_{k \in K} \bar{c}_k (\bar{l}_{k\mu_1} + \bar{l}_{k\mu_2}) \rightarrow \min. \tag{6.20}$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

For each group of production activities considered in the problem a different range of questions is solved: namely, for new enterprises, their location in industrial centres and their production links are determined; for existing enterprises, only their production links are determined; for enterprises of complementary industries, the location of individual plants, or their aggregate by industrial centres, and their production connections are determined.

The links of enterprises of the first group with other enterprises and production facilities through the latter's consumption of their output are reflected through the system of equations (6.1), by which the effect of consumers of output on the choice of operation variant for enterprise j located in sub-areal k is taken into account. In this both intracomplex consumption and shipments out of the complex are considered.

For new enterprises of the specialisation industries the variants of their location in one sub-areal are represented by the separate equations in system (6.1). In that way the indirect connection between groups of variants of the functioning of the enterprises concerned in different sub-areals is realised through consumption of output.

The system of equations (6.1) also reflects the consumption links of functioning enterprises of the specialisation industries. The difference here is simply that each enterprise is represented by one equation since questions of location are not being decided for them. Consequently, with obligatory inclusion of enterprises of the specialisation industries in the composition of a complex [see constraint (6.2)], it is necessary to consider either the utilisation of their output within the complex or its export from the complex.

In contrast to enterprises of the specialisation industries, consumption of the output of complementary industries is considered solely within the complex (6.3) and (6.4). The low transportability of the product, low volume of production at individual enterprises, small differentials between value indicators, and the rigid dependence on the availability of local resources, consumption, and other factors make it possible to assume a limitation on the zone of consumption of certain types of their output.

The distribution of the output of auxiliary and service enterprises within this group of industries, and the demand for their output by enterprises of the specialisation industries, enterprises extracting raw materials, and the public, are indicated in the system of equations (6.4).

In that way the direct effect of variants of the location and operation of enterprises and production activities over the complex is reflected in the model through the system of equations (6.1) and (6.4) when their location scheme is being determined.

When the location scheme of industry is being determined the effect of the choice of different sources of resources is also taken into account. The exploitation of various sources of raw material determines the difference in magnitude of the costs of the functioning of enterprises of the specialisation and complementary industries. Inputs on the operation of industry can be reduced by exploiting the raw material sources with the best extraction and transport conditions and by altering the location of enterprises in relation to raw material sources.

The model considers the possibility of production activities' exploiting of both local and outside raw materials [see (6.3)]. The limits on the development of the extraction industries in a complex, and on the use of outside sources, are determined by their maximum possible extraction from deposits or deliveries from outside sources $(y_{i'p}, y_{i'})$ as well as by the needs of the complex's industry. These limitations are represented in the model being considered by the system of inequalities (6.9) and (6.10).

When the level of development of extractive enterprises based on deposits of the complex under review is being considered, the consumption of each enterprise's output is taken into account separately. This is dictated by the need to allow for the different locations of raw material sources in relation to the consumers, and for the difference in technological methods depending on the quality of the raw material, in accordance with which different modes of operation are envisaged for the enterprises of the specialisation industries and auxiliary production activities. The competition between different raw material sources finds reflection in the comparison of these modes, in particular the competition between local raw materials and those brought into the complex, between different deposits of one type of raw material within the complex, and between sources of interchangeable raw materials.

The transport factor has a great effect on the location of enterprises: namely, the density and configuration of the transport system, its technical equipping, the magnitude of transit traffic across the complex, and other indicators. The effect of the transport factor is accounted for in two ways in the model: namely, (a) through the increase in inputs connected with the appropriate mode of operation of the enterprises, and (b) through the limitation on the carrying capacity of the system's sections. The location of enterprises within the complex in turn can affect the organisation of the region's unified transport system. The direct and feedback connections between the level of development of transport and the location of industry within the complex are taken into account through the system of equations (6.6) and the systems of inequalities (6.7) and (6.8).

Freights connected with the functioning of enterprises of the specialisation, auxiliary, and service industries are considered for each section [see (6.6)]. For that purpose both the existing capacity of the section considered and the additional capacity resulting from different variants of reconstruction must be taken into account. A need to reconstruct sections arises when the total volume of traffic in one exceeds the limitation on its existing carrying capacity, i.e. the value of $\Pi^0_{(kk')}$ in inequalities (6.7). When the value of $\Pi^0_{(kk')}$ is computed, the fact that some of the sections's carrying capacity may be utilised by transit freights across the complex must be allowed for.

Sections of the transport system can be reconstructed in several stages, it being known in advance how much a section's capacity will be increased after one variant of construction or another is implemented. By taking these values as restraints [the value of $\Pi^r_{(kk')}$ in the system of equations (6.8)], the required variant for the reconstruction of each section can be found in the solution of the problem. At the same time the inputs on their reconstruction will be compared with the rise in inputs on the operation of industry under variants that make it possible either to employ only the sections' existing carrying capacity or to reconstruct a minimum number of them. Compromise solutions can also be considered as a result of the comparison.

Expressions (6.6) to (6.8) inclusive describe the transport conditions of the complex in general form. The various types of transport are represented in the model by different sections of the transport system uniting given points within the complex; and the optimum distribution of freight between the types of transport is discovered by means of the targets of the corresponding variants of the operation of enterprises and production activities and by comparing these variants.

The location of enterprises and production activities within a complex presupposes the concentration of a certain amount of labour of the category needed in its various industrial centres. Transfer of population increases inputs on the operation of enterprises through the need to create a set of living conditions in the new place. A need for additional inputs can also arise from the siting of enterprises and production activities in industrial centres that have local reserves of labour, in the case; viz. when there is a rise in the cost of transporting raw materials and fuel, in supplying water and electricity, and in providing services, and so on.

The choice of variant for the siting of enterprises and plants is affected by an increase in inputs on their operation in areas of new construction not only through the additional inputs on providing services for the public but also through the losses involved in underutilisation of various categories of labour.

Three groups of constraints are introduced into the model so as to take these factors into account. The system of equations (6.5) indicates employment differentiated simply by sex and industrial centre. By comparing an industrial centre's total demand for labour of each category with the local reserves of that type we can determine whether it is necessary to bring labour in or whether manpower is being underutilised. This comparison [see equation (6.11)] helps us determine the increment of inputs on operation of the centre's industry entailed by the increase in inputs on providing housing, services, and amenities for the incoming population.

It would not be possible to allow for losses arising from a deviation of the established structure of the labour force from the existing structure of the population by means of constraints (6.5) and (6.11). Breach of these proportions leads to an inability to employ non-employed manpower of one category or another by other units of the economy. In order to find the deviations in the structure of manpower under the different variants of the location of production activities, and so as to allow for the effect of this factor in the location plan, the system of equations (6.13) is introduced.

The efficiency of the location of new specialisation industry enterprises by industrial centres of the complex is substantially determined by the way the area of the centres suitable for industrial and civil building is employed.

The scale of the use of land for industrial and civil construction is represented by the system of inequalities (6.14) and (6.15). The exploitation of another type of local resource, namely water, is represented by the system of inequalities (6.17).

The opportunities for siting enterprises in industrial centres depend to some extent on the size of the latter's resources, the level of inputs on exploiting these resources, and the sanitary-hygienic conditions. The limitations on the use of two of a centre's resources—land and water —are considered in the model [by inequalities (6.16) and (6.18)].

The limitations on sub-areals depend on the area of the various categories of land within them. One category embraces land unsuitable for farming but suitable for building enterprises, housing, and other facilities. As a rule, the inputs on developing it are not great, but its area is limited in extent. The other category consists of areas given over to farming (often very fertile land) or forest. Their conversion calls for considerable inputs to compensate agriculture or to restore forest reserves.

The limitations on water depend on the average daily intake with natural flow of a river, or on regulation of the flow by building water engineering works.

The expedient scale of exploitation of the resources of industrial centres is also determined, to a considerable extent, by the character of the production activities located in the complex. If the siting of some of them in any one centre infringes the sanitary norms (excessive pollution of the atmosphere, effluents, etc.), an unfavourable combination of them in a centre is banned [the system of inequalities (6.19)]. Underutilisation of land and water resources is then possible.

The effect of the main factors on the location of enterprises and plants within a complex is thus taken into account through the system of constraints (6.1) to (6.19) inclusive, and the index of the optimality criterion (6.20).

The size of the aggregate inputs is built up from those on the operation of industries within the complex, the operation and development of transport, development of the area of industrial centres and water supply, and the provision of services for the labour force during its settlement, and also from the losses through underutilisation of various categories of labour.

The model of the spatial structure of a complex does not, of course, resolve the whole range of matters involved in its organisation and functioning and that of its separate parts. Such matters as the process of building a complex in time, organising the system of settlement, and of individual industrial centres, are left unsolved. Their solution calls for the construction of other models, such as those of the spatial structure of an industrial centre (Chapter 9) and of the process of organising a complex (Chapter 10).

7

MODEL OF THE PRODUCTION AND SPATIAL STRUCTURE OF A REGIONAL AGRARIAN-INDUSTRIAL COMPLEX

V. A. Semyonova

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

In a territorial industrial complex with a developed agriculture there must be industries technologically associated with farming. These can be broken down into three groups, as follows:

(1) those processing agricultural raw materials;

(2) those producing supplies for agriculture or providing it with services;

(3) those serving the two foregoing groups.

These industries play a joint socio-economic role in a complex's economy in conjunction with farming. The ultimate goal of their development is to meet the demand of the complex's population for the main food products and to fulfil the plan for shipping foodstuffs and raw materials set at a higher level of planning. The aggregate of the industries technologically and economically associated with farming, and the agriculture of the complex itself, we define as a regional agrarian-industrial complex (RAIC).

Such a RAIC thus includes agriculture, enterprises of the meat, dairy, and food industries, and other enterprises for the primary processing of agricultural raw materials, enterprises for storing, treating, and processing cereals and producing feed concentrates and artificial fertilisers, enterprises for repairing farm machinery, organisations for carrying out land improvement work, and so on.

The specific features of a RAIC's technological and economic links, and the specific factors and conditions of its organisation, call for its singling out as an autonomous part of the investigation and for the constructing of a special model.

The model of a RAIC is closely linked with the other models in the group for organising a territorial industrial complex. The link is to be seen not only in the use that the solutions are put to (plan for the settlement and concentration of population; demand for the RAIC's output) but also in the taking into account of certain limited production resources (land, labour, water). As part of a territorial industrial complex, a RAIC must be treated and analysed in accordance with the general principles for optimising the organisation of the complex as a whole. One must select a rational variant for locating the RAIC's sectors and determining the raw material zones of the industrial enterprises, the volume and direction of transport traffic, the development of the capacities of the plants of all the sectors listed above, and the sources for supplying the population in each administrative district with food products.

Optimisation of a RAIC's production and spatial structure consists in *determining the general plan for the development and location of all its elements that will meet the planned consumption of the region's population for foodstuffs of local origin and meet the target for shipments of the end product, given minimisation of the aggregate total inputs on the building and operation of all its facilities.*

The main functional units of a RAIC are the commercial and intermediate sectors of state-run and co-operative agriculture (both arable farming and stock-raising), the personal smallholdings of country dwellers; the industrial enterprises processing agricultural raw materials; industrial enterprises supplying machinery to agriculture; manpower, the urban and rural population, and the farmland. Industrial and transport links are represented by the flow of produce, agricultural raw materials, and machinery between the RAIC's administrative districts and by imports from outside the RAIC and exports out of it.

The solution of the problem as formulated must take the following factors into account:

(1) the production (technological) links between the various sectors of agriculture and the RAIC's industrial enterprises;

(2) the opportunities for employing different means of developing the material-industrial base and infrastructure of farming;

(3) the permissibility of employing various fodder crops and feedstuffs of different nutritional value in stockraising; the observance of feedstuff consumption rates in terms of fodder quantity and nutrients;

(4) the opportunities for employing various technological means of growing fodder crops and preparing fodder;

(5) the structural features of the "food basket" consumed by the urban and rural population, and the differences in the sources of satisfying the demand for basic foodstuffs;

(6) the limited nature of local resources for developing farm production and of the physical resources needed to extend and intensify farming;

(7) the technological requirements on the ratio of the various types of agricultural raw material delivered for industrial processing.

The field of acceptance of solutions in regard to the objects being studied is decided (a) by the choice of districts for locating the commercial branches of farming and of the technological methods (which are differentiated by the techno-economic indices of the use of the various categories of farmland, by the possibility of developing the material production base of arable farming and stockraising, and by the availability of local resources);

(b) by the choice of areas for siting the RAIC's industrial enterprises and farms producing various types of agricultural raw material and for delivering the finished product to consumers (i.e. to agriculture, and to the urban and rural population);

(c) by the application of various technological means of developing the RAIC's industrial enterprises; and

(d) by the possibility of meeting fodder needs in accordance with the economic expediency of producing various types of concentrated feeds in a district and the conditions for doing so.

The following notation is used in the formal writing of the mathematical economic model.

INDICES AND SETS

i, i' —indices of the resources used by agriculture and by the RAIC's industrial enterprises ($i \in I_1 \cup I_2$, $i' \in I_3 \cup I_4$);

I_1 —set of the local resources of agriculture available in the various administrative districts of the territorial industrial complex at the beginning of the selected year (farmland, labour, farm buildings, etc.);

I_2 —set of the resources and services used in agriculture obtained from the RAIC's industrial enterprises (artificial fertilisers, feed concentrates, repairs, land improvements, schemes, building work, and so on);

I_3 —set of the local resources of the separate administrative districts for intensifying farming (land suitable for amelioration and conversion into more productive farmland);

I_4 —set of resources allocated for the RAIC's purposes (capital investments, water, electricity, etc.);

j —indices of the technological methods of developing the various sectors of the RAIC ($j \in J_1 \cup J_2 \cup J_3 \cup J_4 \cup J_5 \cup J_6$);

J_1 —set of the technological methods for developing the commercial branches of arable farming;

J_2 —set of the technological methods for developing the branches of stockraising;

J_3 —set of the technological methods for producing fodder crops and fodder obtained as a by-product of the commercial branches of farming (straw, chaff, haulms and tops, skim-milk, etc.);

J_4 —set of the technological methods of developing the material production base of farming techniques for improving soil fertility; the building and reconstruction of farm premises, and so on);

J_5 —set of the technological methods for developing the RAIC's industries;

J_6 —set of the technological methods ensuring growth of capacity of the RAIC's industrial enterprises;

s —indices of the commercial sectors of agriculture and personal smallholdings ($s \in S_1 \cup \cup S_2 \cup S_3$);

S_1 —set of the branches of arable farming;

S_2 —set of the branches of stockraising;

S_3 —set of the branches of cultivating personal smallholdings;

l —indices of the administrative districts forming the RAIC ($l \in L$);

L —set of the RAIC's administrative districts;

l' —indices of the RAIC's administrative districts that are consumers of farm produce (foodstuffs) and suppliers of agricultural raw materials ($l' \in L_1' \cup L_2'$);

L_1' —set of the RAIC's administrative districts supplying agricultural raw materials;

L_2' —set of the RAIC's administrative districts consuming food products;

m, m' —indices of the types of agricultural raw material ($m \in M$, $m' \in M$);

M —set of the types of agricultural raw material;

k —indices of the types of fodder ($k \in K$);

K —set of the types of fodder ($K \cap I_2 = K'$);

k' —sub-set of the types of concentrates ($k' \in K'$; $K' \subset I_2$);

t —indices of the types of nutrient ($t \in T$);

T —set of the types of nutrient;

h —indices of the food products produced by the RAIC ($h \in H_1 \cup H_2$);

H_1 —set of the food products processed in the RAIC's industrial enterprises;

H_2 —set of food products passing into consumption without processing;

r —indices of the RAIC's industrial enterprises ($r \in R_1 \cup R_2$);

R_1 —set of the sectoral groups of the RAIC's industrial enterprises processing agricultural raw material;

R_2 —set of the sectoral groups of industrial enterprises supplying machinery and services to agriculture;

n —indices of the population groups of the territorial industrial complex ($n \in N$; $N = \{1,2\}$, $n = 1$—urban population, $n = 2$—rural population).

MATRIX COEFFICIENTS

b_{ijsl} —input of type i resource per unit of the rate of technological method j of developing branch s of commercial farming in district l;

b_{ijkl} —input of i-type resource per unit of intensity of technological method j of producing fodder k in district l;

p_{ijsl} —input of resource i obtained from the RAIC's industrial enterprises per unit of intensity of technological method j of developing commercial branch s of farming in district l;

p_{ijkl} —input of resource i obtained from the RAIC's industrial enterprises per unit of intensity of technological method j of producing fodder k in district l;

b_{ijl} —increment of local agricultural resource i ensured by a unit of intensity of technological method j in district l;

$b_{i'ijl}$ —input of resource i' per unit of technological method j of increasing farmland, manpower, farm buildings, and the capacities of the RAIC's industrial enterprises, etc. (i.e. of i-type resources) in district l;

$b_{i'rjl}$ —input of resource i' per unit of technological method j of developing industrial enterprise r of the RAIC in district l;

a_{tjsl} —annual demand for nutrient t per unit of intensity of technological method j of developing commercial sector s of state-run and co-operative stockraising in district l;

a_{tsl} —annual demand for nutrient t per unit of intensity of the development of sector s of the cultivation of personal smallholdings in district l;

a_{tjkl} —quantity of nutrient t contained in a unit of fodder k produced by technological method j in district l;

$a_{tkrl'}$ —quantity of nutrient t contained in a unit of fodder k obtained from feed-concentrate factory r located in district l';

$\underline{\alpha}_{kjsl}$ and $\overline{\alpha}_{kjsl}$—minimum and (corresponding) maximum annual norm for consumption of fodder k per unit of intensity of technological method j of developing commercial branch s of state-run and co-operative stockraising in district l;

$\underline{\alpha}_{ksl}$ and $\overline{\alpha}_{ksl}$ —minimum and (corresponding) maximum annual input of fodder k per unit of intensity of the development of sector s of the cultivation of personal smallholdings in district l;

α_{kjl} —quantity of fodder k provided by a unit of intensity of technological method j of fodder production in district l;

$\alpha_{krll'}$ —quantity of fodder k provided by a unit of intensity of deliveries by industrial enterprise r from district l' to district l;

γ_{hnl} —annual standard of consumption of food product h by group n of the population of district l;

γ_{hrl} —annual volume of production of food product h per unit of capacity of industrial enterprise r in district l;

p_{irl} —annual volume of production of resource i per unit of capacity of industrial enterprise r located in district l;

p_{krl} —annual volume of production of fodder k per unit of capacity of industrial enterprise r located in district l;

p_{hjsl} —quantity of food product h provided by a unit of intensity of technological method j of developing commercial branch s of agriculture in district l;

p_{hsl} —quantity of food product h provided by a unit of intensity of the development of sector s of the cultivation of personal smallholdings in district l;

p_{mjsl} —quantity of raw material (type m) provided by a unit of intensity of technological method j of developing commercial sector s of agriculture in district l;

p_{mrl} —annual norm of inputs of m-type agricultural raw material per unit of capacity of industrial enterprise r located in district l;

p_{mhl} —coefficient of the conversion of food product h, produced in district l and supplied for consumption without industrial processing, into an m-type agricultural raw material;

$u_{mm'rll'}$ —quantity of agricultural raw material m going into a unit of raw material m' delivered from district l' for processing in industrial enterprise r located in district l.

CONSTRAINTS

B_{il}, $\bar{B}_{i'l}$ —availability of resources i and i' in district l;

$B_{i'}$ —limit on the use of resource i' by the RAIC as a whole;

P_{rl} —magnitude of the previously built capacity of industrial enterprise r in district l;

P_h —plan for the shipment of food product h out of the RAIC;

P_m —plan of the shipment of agricultural raw material (type-m) out of the RAIC;

B_{nl} —forecast population of group n in district l.

FUNCTION DESIGNATORS

c_{jsl} —annual current input per unit of intensity of technological method j of developing branch s of agriculture in district l;

c_{kjl} —annual current inputs per unit of intensity of technological method j of producing fodder k in district l;

c_{rl} —annual current inputs on exploiting a unit of capacity of industrial enterprise r in district l;

c_{ijl} —normalised inputs per unit of intensity of technological means j ensuring growth of resource i in district l;

c_{rjl} —normalised inputs per unit of intensity of technological means j of increasing the capacity of industrial enterprise r in district l;

$c_{mrll'}$ —normalised inputs on transporting a unit of m-type agricultural raw material from district l' to industrial enterprise r located in district l;

$c_{hrll'}$ —normalised inputs on transporting a unit of food product h to district l' from industrial enterprise r located in district l;

$c_{krll'}$ —normalised inputs on transporting a unit of fodder k to district l from industrial enterprise r located in district l';

c_{hrl} —normalised input on transporting a unit of food product h from industrial enterprise r in district l to the shipping point for sending it out of the RAIC;

c_{ml} —normalised input on transporting a unit of m-type agricultural raw material from district l to the shipping point for sending it out of the RAIC;

UNKNOWNS

x_{jsl} —rate of exploitation of technological method j of developing sector s of agriculture in district l;

x_{kjl} —rate of exploitation of technological method j of producing fodder k in district l;

x_{sl} —intensity of the development of branch s of the cultivation of personal smallholdings in district l;

x_{rl} —capacity of industrial enterprise r in district l;

x_{ijl} —intensity of the exploitation of technological method j ensuring growth of resource i in district l;

x_{rjl} —intensity of the exploitation of technological method j of increasing the capacity of industrial enterprise r in district l;

x_{nl} —size of population group n in district l;

$y_{krll'}$ —volume of deliveries of fodder k to district l' from industrial enterprise r located in district l;

y_{hnl} —quantity of food product h sold by socially-owned agriculture for consumption by population group n of district l;

y_{hrl} —quantity of food product h shipped out of the RAIC from industrial enterprise r located in district l;

y_{ml} —amount of agricultural raw material, type m, shipped out of the RAIC from district l;

$y_{mrll'}$ —volume of freights of agricultural raw material, type m, from district l' to industrial enterprise r in district l;

$y_{hrll'}$ —volume of deliveries of food product h from industrial enterprise r located in district l, for the population of district l'.

The model for optimising the production and spatial structure of a regional agrarian-industrial complex is as follows:

Find the non-negative values of x_{jsl}, x_{kjl}, x_{sl}, x_{rl}, x_{ijl}, x_{rjl}, x_{nl}, $y_{krll'}$, y_{hnl}, y_{hrl}, y_{ml}, $y_{mrll'}$, and $y_{hrll'}$, such as will satisfy the constraints given below.

The public's demand in each district for food products obtained after processing in the RAIC's industrial enterprises is satisfied:

$$\sum_{n\in N} \gamma_{hnl}x_{nl} - \sum_{r\in R_1} \sum_{l'\in L_2'} y_{hrll'} \leqslant 0 \quad (h\in H_1,\ l\in L). \qquad (7.1)$$

The urban population of each district is ensured food products entering consumption without industrial processing:

$$\gamma_{hnl}x_{nl} - \sum_{j\in J_1} \sum_{s\in S_1} p_{hjsl}x_{jsl} \leqslant 0 \quad (h\in H_2,\ l\in L,\ n=1). \qquad (7.2)$$

The rural population's demand for food products not requiring industrial processing is met from personal smallholdings and socially-owned production:

$$\gamma_{hnl}x_{nl} - \sum_{s\in S_3} p_{hsl}x_{sl} - y_{hnl} \leqslant 0 \ (h\in H_2,\ l\in L, n=2). \qquad (7.3)$$

The whole forecast population (urban and rural) is taken into account when the consumption of food products in each district is determined:

$$x_{nl} \geqslant B_{nl} \quad (n\in N,\ l\in L). \qquad (7.4)$$

The RAIC meets the plan for deliveries of:

(a) finished products

$$\sum_{r\in R_1} \sum_{l\in L} y_{hrl} \geqslant P_h \quad (h\in H_1 \cup H_2); \qquad (7.5)$$

(b) agricultural raw materials

$$\sum_{l\in L} y_{ml} \geqslant P_m \quad (m\in M). \qquad (7.6)$$

The scale of development of agriculture in each district is limited by the existing local reserves:

$$\sum_{s\in S_1\cup S_2}\sum_{j\in J_1\cup J_2} b_{ijl}x_{jsl}+\sum_{k\in K}\sum_{j\in J_3} b_{ikjl}x_{kjl}-\sum_{j\in J_4} b_{ijl}x_{ijl}\leqslant B_{il}$$
$$(i\in I_1,\ l\in L). \qquad (7.7)$$

The opportunities for intensifying farming in each district are limited by the availability of the appropriate local resources:

$$\sum_{j\in J_4}\sum_{i\in I_1} b_{i'ijl}x_{ijl}\leqslant \overline{B}_{i'l}\quad (i'\in I_3,\ l\in L). \qquad (7.8)$$

A fodder base is organised in each district that is balanced for nutrients and the appropriate feeding requirements of animal husbandry and for the long-term technological means of developing stockraising and the fodder base:

(a) input-output matrices for nutrients

$$\sum_{j\in J_2}\sum_{s\in S_2} a_{tjsl}x_{jsl}+\sum_{s\in S_3} a_{tsl}x_{sl}-\sum_{k\in K}\sum_{j\in J_3} a_{tkjl}x_{kjl}-$$
$$-\sum_{r\in R_2}\sum_{k\in K}\sum_{l'\in L_1'} a_{tkrl'}y_{krll'}\leqslant 0\quad (t\in T,\ l\in L); \qquad (7.9)$$

(b) input-output matrices for separate types of fodder

(i) by minimum norms of feeding

$$\sum_{j\in J_2}\sum_{s\in S_2} \underline{\alpha}_{kjsl}x_{jsl}+\sum_{s\in S_3} \underline{\alpha}_{ksl}x_{sl}-$$
$$-\sum_{j\in J_3} \alpha_{kjl}x_{kjl}-\sum_{r\in R_2}\sum_{l'\in L_1'} \alpha_{krll'}y_{krll'}\leqslant 0\quad (k\in K,\ l\in L); \qquad (7.10)$$

(ii) by maximum norms of feeding

$$\sum_{j\in J_2}\sum_{s\in S_2} \overline{\alpha}_{kjsl}x_{jsl}+\sum_{s\in S_3} \overline{\alpha}_{ksl}x_{sl}-\sum_{j\in J_3} \alpha_{kjl}x_{kjl}-$$
$$-\sum_{r\in R_2}\sum_{l'\in L_1'} \alpha_{krll'}y_{krll'}\geqslant 0\quad (k\in K,\ l\in L). \qquad (7.11)$$

The volume of deliveries of agricultural raw materials from each district and the consumption by the rural population are balanced with the scale of production in that district:

$$\sum_{j\in J_1\cup J_2}\sum_{s\in S_1\cup S_2} p_{mjsl}x_{jsl}-y_{ml}-\sum_{l'\in L_1'}\sum_{r\in R_1} y_{mrll'}-$$
$$-\sum_{h\in H_2}\sum_{n=2} p_{mhl}y_{hnl}\geqslant 0 \quad (m\in M,\ l\in L). \qquad (7.12)$$

The scale of development of the individual enterprises for producing foodstuffs is adequate to satisfy the demand of the complex's population and the plan for shipments out of the complex:

$$\gamma_{hrl}x_{rl}-\sum_{l'\in L_2'} y_{hrll'}-y_{hrl}\geqslant 0 \quad (r\in R_2,\ h\in H_1,\ l\in L). \qquad (7.13)$$

The scale of development of industrial enterprises for processing agricultural raw materials is ensured by the appropriate deliveries:

$$p_{mrl}x_{rl}-\sum_{l'\in L_1'} y_{mrll'}\leqslant 0 \quad (m\in M,\ r\in R_1,\ l\in L). \qquad (7.14)$$

Definite ratios are maintained between the separate types of raw material delivered for processing:

$$\sum_{l'\in L_1'} y_{m'rll'}-\sum_{m\in M\setminus m'}\sum_{l'\in L_1'} u_{mm'rll'}y_{mrll'}\geqslant 0$$
$$(m'\in M,\ r\in R_1,\ l\in L). \qquad (7.15)$$

The capacity of the RAIC's industrial enterprises producing physical resources and servicing agriculture is adequate to the needs of arable farming and stockraising in all districts of the complex for each type of product or service (except feed concentrates):

$$\sum_{r\in R_2}\sum_{l\in L} p_{irl}x_{rl}-\sum_{l\in L}\sum_{s\in S_1\cup S_2}\sum_{j\in J_1\cup J_2} p_{ijsl}x_{jsl}-$$
$$-\sum_{l\in L}\sum_{k\in K}\sum_{j\in J_3} p_{ikjl}x_{kjl}\geqslant 0,$$
$$(i\in I_2\setminus k'). \qquad (7.16)$$

The capacity of industrial enterprises producing feed concentrates is adequate to meet the needs of a district of the complex:

$$p_{krl}x_{rl}-\sum_{l'\in L_1'} y_{krll'}\geqslant 0 \quad (k\in K',\ r\in R_2,\ l\in L). \qquad (7.17)$$

The input-output matrices of the productive capacity of enterprises are:

$$x_{rl}-\sum_{j\in J_6} x_{rjl}\leqslant P_{rl} \quad (r\in R_1\cup R_2,\ l\in L). \qquad (7.18)$$

The input of resources needed to intensify agriculture and develop the RAIC's industry do not exceed the limit imposed for these purposes:

$$\sum_{l\in L'_2}\sum_{j\in J_4}\sum_{i\in I_1} b_{i'ijl}x_{ijl}+\sum_{j\in J_5}\sum_{r\in R}\sum_{l\in L} b_{i'rjl}x_{rjl}\leqslant B_{i'} \quad (i'\in I_4). \qquad (7.19)$$

In observing constraints (7.1) to (7.19) inclusive the aggregate inputs on building, developing, and operating all the RAIC's facilities must be minimal:

$$\sum_{l\in L}\sum_{j\in J_1\cup J_2}\sum_{s\in S_1\cup S_2} c_{jsl}x_{jsl}+\sum_{l\in L}\sum_{j\in J_3}\sum_{k\in K} c_{kjl}x_{kjl}+\sum_{l\in L}\sum_{r\in R} c_{rl}x_{rl}+$$
$$+\sum_{l\in L}\sum_{i\in I}\sum_{j\in J_4} c_{ijl}x_{ijl}+\sum_{l\in L}\sum_{j\in J_6}\sum_{r\in R} c_{rjl}x_{rjl}+$$
$$+\sum_{l\in L}\sum_{l'\in L'_1}\sum_{m\in M}\sum_{r\in R_1} c_{mrll'}y_{mrll'}+\sum_{l\in L}\sum_{l'\in L'_1}\sum_{k\in K}\sum_{r\in R_2} c_{krll'}y_{krll'}+$$
$$+\sum_{l\in L}\sum_{l'\in L'_2}\sum_{h\in H_2}\sum_{n\in N} c_{hnll'}y_{hnll'}+\sum_{l\in L}\sum_{r\in R_1}\sum_{h\in H_1} c_{hrl}y_{hrl}+$$
$$+\sum_{l\in L}\sum_{m\in M} c_{ml}y_{ml}\rightarrow \min. \qquad (7.20)$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

One of the main problems to be solved in the organising of a RAIC is that of determining the volume of production of foodstuffs and agricultural raw materials. The need for food products obtained by processing agricultural raw materials in industrial enterprises is determined by the differentiated norms of their consumption by the urban and rural population of each administrative district on the basis of the system of inequalities (7.1), which makes it possible at the same time to establish the suppliers of these products (districts and enterprises) and to bring out the links within the complex for their production and consumption.

When the demand for food products entering consumption without industrial processing is being determined, the differences between the sources for satisfying this demand are taken into account in addition to the differentiation between the consumption standards for the urban and rural population. The model envisages that the urban population's demand for foods of this kind will be met by the commercial branch of agriculture (7.2). For that purpose the consumption norms allow for townspeople's consumption of such things as potatoes and vegetables to be reduced by the quantities they obtain from private and collective kitchen gardens. As for the rural population, two sources of foodstuffs not requiring industrial processing are envisaged: namely, personal smallholdings and the market branches of state and co-operative farming (7.3).

The basis for determining the complex's consumption of both types of foodstuffs, together with the consumption norms, is the forecast of urban and rural population in each district (7.4).

The targets for deliveries of foodstuffs outside the complex are allocated among enterprises and districts by the inequalities (7.5), while constraints (7.13) tie up the schemes for supplying its population with industrially processed food products and for meeting outside delivery targets, with the capacity of the processing enterprises.

The system of inequalities (7.1) to (7.5) inclusive and (7.13) makes it possible to localise demand for, and volume of production of foodstuffs in a breakdown of the individual administrative districts and industrial enterprises.

Similar matters relating to the siting of production activities and consumption of agricultural raw materials are resolved by the system of inequalities (7.6), (7.12), (7.14), and (7.15). The first two constraints co-ordinate the RAIC's targets for export of raw materials with the location of their production by separate districts, at the same time allowing for their consumption in its own enterprises and by the local rural population. The other two constraints give precision to the structure of intracomplex links for agricultural raw materials, ensuring correspondence of the capacities for processing the separate types of raw material in each of the industrial enterprises with the scale of deliveries, and satisfying the technological conditions for processing them in accordance with their different types.

The organisation of the fodder base of stockraising is reflected in the model by constraints (7.9) to (7.11) inclusive and (7.17). The scale of development and structure of the fodder base of each district, taking interdistrict links into account, ensure meeting of the animal husbandry requirements for feeding cattle and poultry, i.e. balancing of nutrients (7.9) and adequate variety of feedstuffs (7.10) and (7.11). The possibility of establishing local norms for inputs of fodder within the limits of their minimum and maximum values offered by constraints (7.10) and (7.11) makes it possible, on the one hand, to ensure a balance of nutrients and, on the other hand, to organise the local fodder base allowing for the peculiarities of the conditions for producing the various types of fodder in each district. In addition, these inequalities, in connection with constraints (7.17), provide the basis for determining the capacities and locations of industrial enterprises for producing concentrated feeds, which constitute an important part of the fodder base for developing stockraising.

The industrial enterprises producing various types of physical resource for agriculture and servicing it must meet each type of these needs for all branches of commercial arable farming, stockraising, and the RAIC's fodder base. This requirement is met in the mathematical model by the system of inequalities (7.16), which enables the scale of demand for each type of resource $(i \in l_2 \setminus k')$ to be established and the scale of the appropriate enterprises and their location in the complex to be determined.

The spatial connections between the RAIC's districts as regards the use and export of various kinds of produce are represented by the modes of production and transport. Their intensity is expressed in the volumetric mensuration units of the production of the relevant output, i.e. in tonnes, quintals, cubic metres, and so on [see constraints (7.1), (7.3), (7.5), (7.6), and (7.9) to (7.17) inclusive].

In addition to reflecting spatial connections, it is necessary to allow for the intersectoral connections of the RAIC's facilities, since all sectors of the complex's economy are being developed in an interconnected way in definite proportions with one another. An increase or decline in the volume of production of one sector or another gives rise to corresponding changes in the development of the others. This interconnection and reciprocal conditioning of the development of all branches of the RAIC's economy is ensured in the model by the technological means of developing the complex's separate facilities.

Each technological method has its own units of measurement and intensiveness, indices of output, demand for resources of one kind or another, and inputs on operating or effecting the method.

The units for measuring the intensity of technological method are as follows: for arable farming, the hectare of sown area; for stockraising, the number of head of the main herd, including the young animals; for the production of fodder crops, the tonne or quintal of physical weight; for industrial enterprises and other industrial and building facilities, their capacities in standard output or services. The characteristics of the intensiveness of a technological method thus reflect the separate nature of each sector, enterprise, or production activity, and make it possible to employ the information used by industrial and agricultural planning authorities when solving the problem.

Each technological method is characterised by indices of the output of its appropriate product, of the need in the output of other sectors (enterprises, production activities) and in resources. Their use also ensures reciprocal interconnections in the development of the RAIC's various facilities and observance of the limits on inputs both of limited local (district) resources, and of resources common to the whole complex. The scale of the different methods of improving soil fertility, and other ways of intensifying agriculture, are limited by the availability of the appropriate resources in each district [constraint (7.8)]. The particular local resources for developing the RAIC's industry are the already existing capacities of the industrial enterprises of each district, employment of which for the complex's future economic development must be confined to a scale not exceeding their actual availability [see constraint (7.18)]. In addition the total scale of the development of agriculture and industry in all districts must correspond to given limits common for the whole set of resources ($i' \in I_4$), which is reflected in inequalities (7.19).

The technological and industrial-transport methods of the functioning and development of a RAIC also have indices of the inputs needed to realise them, which in aggregate forms the objective function (7.10), which expresses the requirement of minimising normalised inputs on agricultural produce, the processing of raw material, and the production of artificial fertilisers, fodder, and other physical resources and services by the RAIC's facilities, on transporting the raw material or the finished product, and on intensifying farming and developing the complex's industry.

In this way the technological and industrial-transport methods link together the whole system of constraints of the optimum functioning of the RAIC's industrial and spatial structure, so as to ensure its balanced development.

8

MODEL OF THE LOCATION OF A COMPLEX'S INDUSTRY

V. I. Ezersky

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

The problem of optimising the location of industrial production activities within a single complex consists in *choosing such variants of the development of industrial centres* (their structure, and the scale of development of industrial enterprises), *the composition and direction of industrial connections, the exploitation and development of main-line transport, that their targets for production of the types of output envisaged will be met with minimum inputs*, taking into account provision of the assigned standard of living of the centres' population, satisfying of the technological conditions for the operation of the activities concerned, and the requirements of industry's territorial organisation.

The following notation is used to write the mathematical model.

INDICES AND SETS

i, j —indices of the complex's industrial centres ($i \in I$);

I —the set of industrial centres;

l —index of a product or resource ($l \in L$);

k —index of a consumer or supplier located outside the complex ($k \in K_1$, $k \in K_2$);

K_1 —the set of suppliers;

K_2 —the set of consumers;

r —index of the variants of the development of industrial centres;

R_i —the set of variants for industrial centre i;

q —index of sections of the transport system ($q \in Q$);

Q —set of the sections of the transport system within the complex;

f —index of the variants of the development of sections of the transport system ($f \in F_q$);

F_q —set of the variants of development of section q of the transport system.

MATRIX COEFFICIENTS

A_i^{lr} —volume of production of product (resource) l by variant r of the development of industrial centre i;

B_i^{lr} —consumption of product (resource) l by industrial centre i under development variant r;

D_q^f —carrying capacity of section q of the transport system under development variant f;

β^l —freight volume of l-type product;

δ_{ij}^q —coefficient determining the degree of use of section q of the transport system for freight traffic by the appropriate route: $\delta_{ij}^q = 1$ when the route from i to j passes along section q; $\delta_{ij}^q = 0$ in the contrary case (similarly $\delta_{ki}^q = 0; 1$ and $\delta_{ik}^q = 0; 1$).

CONSTRAINTS

N_q —volume of transit freight traffic over section q of the transport system;

G_k^l —target for shipments of product (resource) l out to consumer k;

H_k^l —target for shipments of product (resource) l in from supplier k.

FUNCTION DESIGNATORS

c_i^r —adjusteds costs of implementing variant r of the development of industrial centre i;

s_{ij}^l, (s_{ki}^l, s_{ik}^l) —unit input on transporting a unit of product (resource) l by routes $(i—j)$, $(k—i)$, $(i—k)$;

v_q^f —adjusted costs of carrying out variant f of the development of section q of the transport system.

VARIABLES

x_i^r —rate of variant r for the development of industrial centre i;

y_{ij}^l —delivery of product (resource) l produced in industrial centre i to industrial centre j;

y_{ki}^l —delivery of product (resource) l from supplier k to industrial centre i;

y_{ik}^l —delivery of product (resource) l from industrial centre i to consumer k;

z_q^f —rate of variant f of the development of section q of the transport system.

We write down the mathematical model of the problem as follows:

Find $x_i^r \geqslant 0$; $y_{ij}^l \geqslant 0$; $y_{ki}^l \geqslant 0$; $y_{ik}^l \geqslant 0$; $z_q^f \geqslant 0$ that will satisfy the following constraints.

An obligatory distribution of product l produced in industrial centre i is considered:

$$\sum_{r\in R_i} A_i^{lr} x_i^r - \sum_{j\in I\setminus i} y_{ij}^l - \sum_{k\in K_2} y_{ik}^l = 0 \quad (i\in I;\ l\in L). \qquad (8.1)$$

The demand of centre i's industries for product (resource) must be met:

$$-\sum_{r\in R_i} B_i^{lr} x_i^r + \sum_{j\in I\setminus i} y_{ij}^l + \sum_{k\in K_1} y_{ki}^l \geqslant 0 \quad (i\in I,\ l\in L). \qquad (8.2)$$

The load on a section of the transport system of the complex is built up allowing for transit traffic and the possible carrying capacity:

$$-\sum_{i\in I}\sum_{l\in L}\sum_{j\in I\setminus i}\beta_l\delta^q_{ij}y^l_{ij}-\sum_{i\in I}\sum_{l\in L}\sum_{k\in K_1}\beta_l\delta^q_{ki}y^l_{ki}-$$
$$-\sum_{i\in I}\sum_{l\in L}\sum_{k\in K_2}\beta_l\delta^q_{ki}y^l_{ki}+\sum_{f\in F_q}D^f_q z^f_q\geqslant N^f_q \quad (q\in Q). \qquad (8.3)$$

Targets for the delivery of products (resources) outside the complex must be met:

$$\sum_{i\in I} y^l_{ik}\geqslant G^l_k \quad (l\in L;\ k\in K_2). \qquad (8.4)$$

The limitations on the import of products (resources) into the complex must be observed:

$$\sum_{i\in I} y^l_{ki}\leqslant H^l_k \quad (l\in L;\ k\in K_1). \qquad (8.5)$$

The following choice of development variant is envisaged: (i) for industrial centres

$$\sum_{r\in R_i} x^r_i\leqslant 1 \quad (i\in I); \quad x^r_i=\begin{cases}0\\1\end{cases} \qquad (8.6)$$
$$\text{(for all } i \text{ and } r).$$

(ii) for sections of the transport system

$$\sum_{f\in F_q} z^f_q\leqslant 1 \quad (q\in Q); \quad z^f_q=\begin{cases}0\\1\end{cases} \qquad (8.7)$$
$$\text{(for all } q \text{ and } f).$$

The grand total of the aggregate adjusted costs of the organising and functioning of the complex's economy must be minimal:

$$\sum_{i\in I}\sum_{r\in R_i} c^r_i x^r_i+\sum_{i\in I}\sum_{l\in L}\sum_{j\in I\setminus i} s^l_{ij}y^l_{ij}+$$
$$+\sum_{i\in I}\sum_{l\in L}\sum_{k\in K_1} s^l_{ki}y^l_{ki}+\sum_{i\in I}\sum_{l\in L}\sum_{k\in K_2} s^l_{ik}y^l_{ik}+\sum_{q\in Q}\sum_{f\in F_q} v^f_q z^f_q\rightarrow \min. \qquad (8.8)$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

Several possible industrial centres are distinguished within the complex being considered, linked together by transport lines, power lines, and other communications. Several development variants are considered for each centre, differing in the composition, scale, and specialisation of industrial enterprises, sources of supply of resources of sectoral significance, and zones of consumption of their output. The existence of centres with only one enterprise is permitted.

Both existing and planned centres are treated in the model.

In the plan period it is possible to reconstruct the economy of existing centres without adding new enterprises of the specialisation industries in order (a) to extend production of individual products, (b) to improve the provision of housing, cultural institutions, and amenities, (c) to improve sanitary and hygienic conditions, and so on. The variants of reconstruction may differ from one another in the capacity of enterprises and the number of people in settlements, the set of auxiliary and service industries, the variants for reconstructing a centre's system of municipal engineering works and communications, and so on. It is also possible to build new facilities in existing industrial centres.

The variants for the development of new centres provide for a combination of industrial enterprises, people's settlements, and engineering and transport communications in the appropriate area, that call for a certain quantity of resources, produce a given set of products, and are characterised by capital and current costs of their building and functioning.

Each variant of a centre must meet the sanitary, hygienic, and other territorial planning requirements laid down for it. The limitations on the industrial and built-up area and water resources of any one sub-areal are taken into account in the forming of a centre; and variants are considered for the treatment and disposal of effluents, the features of the area's microclimate are taken into account, and so on.

So that the selection of centre's development variants will satisfy the requirement of proportionality in the development of interconnected industries, the input-output matrix of the production and distribution of output (8.1) is considered in the model. The definition of the set of products and resources depends on the production structure of the planned complex's economy. Two identical products produced by different enterprises of a centre are treated as different products. The system of equations (8.2) represents the constraints on satisfying the demand of centres for needed products and resources.

The carrying capacity of transport communications within a complex, and the magnitude of capital and operation costs, depend on their development variants. The transport routes of all products (resources) are planned according to the appropriate sections of the system. The model's constraints (8.3) mean that the total volume of freight carried by sections of the transport system must not exceed their carrying capacity under the appropriate development variants of these sections. The amount of transit traffic is taken into account through the introduction of a requirement for transit traffic on the right-hand side of constraint (8.3).

A separate group of constraints characterises the complex's main external economic links with other units of the national economy. Thus constraints (8.4) reflect the target for shipping certain products and resources, produced mainly by the complex's specialisation industries, out of the complex, while constraint (8.5) reflects the possibility of bringing in certain products and resources of regional and inter-regional significance (capital investments, labour, certain scarce types of output).

The optimality criterion of the problem reflects the requirement of minimising aggregate total inputs on creating and operating the complex's economy.

The function designators include the normalised inputs by development variants of the industrial centres and sections of the transport system, and the normalised unit transport inputs.

The adjusted costs of developing a separate centre (c_i^r) reflect the outlays on engineering preparation and area development, including the specialisation, auxiliary, and service industries, expenditure on building and operating housing, cultural and public utilities, and amenities, and the inputs on municipal engineering works of importance for the whole centre. These data are taken from the drafts of the project and design institutes concerned.

The normalised inputs on developing an individual centre are calculated by means of the following formula:

$$c_i^r = E\,[\sum_p K_i^{rp} + K_{i(\mathrm{tp})}^r + K_{i(\mathrm{cent})}^r] + \sum_p N_i^{rp} + N_{i(\mathrm{tp})}^r + N_{i(\mathrm{cent})}^r,$$

in which K_i^{rp} —is the capital investment on building enterprise p in an industrial centre i under variant r of the centre's development;

$K_{i\,(tp)}^r$ —is the capital investment on the town planning and building of industrial centre i under variant r of its development;

$K_{i\,(cent)}^r$ —is the capital investment of centre i's auxiliary and service economy and its municipal engineering public works and communications; and r variant of its development;

N_i^{rp} —is the costs of production of industrial enterprise p under variant r of its development in industrial centre i less the cost of raw materials and supplies;

$N_{i\,(tp)}^r$ —is the currentcosts of housing and public utilities and institutions serving the public living in industrial centre i, under variant r of its development;

$N_{i\,(cent)}^r$ —is the current costs of exploiting centre' i's auxiliary and service economy and its systems and public works.

The adjusted costs of developing the sections of the transport system (v_q^f) and the unit costs of transporting products and resources by traffic routes (s_{ij}^l, s_{ki}^l) are interdependent and are calculated by the appropriate norms, allowing for the conditions of the locality through which the trunk transport lines pass, and the factors raising the cost of building them. Railways, roads, water transport, pipelines, and other forms of intercentre transport are considered in accordance with their significance. The indices v_q^f are calculated for all possible variants of the development of each section of the transport system of the relevant type of transport. The indices s_{ij}^l and s_{ki}^l are calculated for each traffic route, and differentiated by type of freight.

The solution of the problem of the location of industry has to be analysed; in particular the development plan and the scheme for the location of enterprises in a sectoral breakdown, the organisation plans and settlement scheme for the various industrial centres, the scheme for the location of elements of the complex's economy as a whole, and the plan for the exploitation and development of trunk transport lines are examined in detail.

The aim of the analysis in sectoral break-down is to compare the efficiency of facilities of the same type, identify those for immediate construction, investigate sites, clarify the reasons dictating this choice, and so on. Very close attention is paid in this to the main specialisation industries of the complex's economy.

The location plan obtained for the specialisation industries can be analysed by comparing (a) all the variants of their location and development either given or introduced into the optimum plan, and (b) the various schedules of input involved by these variants. The aim of analysing an industrial centre's development plan is to clarify the factors most affecting choice of its industrial structure. This is done by comparing the conditions (factors) of the operation of enterprises and of the settlement of workers in the territories (sub-areals) of the centres, by virtue of which there are higher or lower outlays on the complex as a whole. The shadow prices of output and consumption are used for this purpose.

The aim of analysing the location plan of a complex's economy as a whole is to make a general analysis of the solution obtained, which must reflect the location and scale of development of industrial enterprises on its territory, the system and the size of settlements, the development of the transport system and of water mains, sewerage, etc., and the composition, intensity, and direction of the production links of enterprises and industrial centres.

9

MODEL OF AN INDUSTRIAL CENTRE'S SPATIAL STRUCTURE

V. S. Zverev and V. Yu. Malov

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

Industrial centres and agglomerations are the main territorial industrial combinations within a complex. By an industrial centre we mean the aggregate of sites of production activities and population united by a common production and social infrastructure. A concentration of several industrial centres in a limited area forms an agglomeration. The location of the main industrial centres of a territorial industrial complex and their industrial specialisation and external connections are determined by means of the model of a complex's spatial structure. But at that stage of the investigation such matters as the location of enterprises of a centre's specialisation industries by sub-areals, and their combination in a sub-areal, the scale of development and location of the facilities of the production and social infrastructure of intracentre significance, the distribution of population within a centre, the functional zoning of a centre and the distinguishing of industrial and built-up areas and green zones, recreation spots, and so on, have not been decided. Thus, when a centre's spatial structure is being modelled, the problem is *to determine the variant of the functional zoning of its area, the distribution of enterprises by individual sub-areals and their combinations within sub-areals, the development of the production and social infrastructure, the distribution of population that will ensure minimum inputs, while observing its planned living conditions, on the building and functioning of the industrial enterprises* and facilities of the centre and of the production and social infrastructure, on municipal amenities and public transport.* A *sine qua non* of this is the meeting of output targets by the centre's enterprises.

Solving of the problem as formulated requires taking the following factors into account:

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* Only those enterprises are considered that determine a centre's industrial structure. Any others will be covered by "facilities".

(a) the target for setting the production capacities of the enterprises in the centre;

(b) the connections between enterprises in joint exploitation of resources, the centre's municipal engineering services, and auxiliary facilities of local significance;

(c) the conditions for the distribution and employment of each category of labour by settlements and spheres of activity;

(d) the conditions for the building and development of the centre's municipal engineering services and auxiliary facilities of local importance, and for the exploitation of resources;

(e) the conditions for organising the centre's population settlements.

For that purpose corresponding constraints (equations and inequalities) are introduced into the model. The effect of these factors can be taken partially into account by the designators of the objective function, for which the minimum adjusted costs of the functioning of the centre as a whole serve as the indices.

The objects of modelling in optimising a centre's spatial structure are the combinations of new and existing enterprises, separate enterprises or groups of jointly located existing enterprises, sections of the centre's system of municipal engineering services and the municipal facilities associated with it, the access railway lines to sub-areals, the sectors of the sub-areals, the scale of commuting, population settlements, and the manpower reserves.

The field of acceptance of solutions for these objects is decided by the possibility of selecting sites for locating and combining enterprises within a sub-areal, of utilising various elements of the infrastructure for all enterprises, the scale of exploitation of the carrying capacity of sections of the centre's municipal engineering services and the capacities of the facilities associated with them, the shape and degree of exploitation of the access ways to sub-areals, the scale of utilisation of the area for various sub-areals, the number of workers commuting to various work places. The formulation of the problem proposed is embodied in a mathematical economic model, for the formal writing of which the following notation is used.

INDICES AND SETS

j —index of an enterprise of the specialisation industries $(j \in J)$;

J —set of the enterprises of a centre's specialisation industries;

k —index of sectors (and access ways) of the sub-areal of a centre;

K —set of the sectors within the centre being studied;

$\overline{K}$ —set of the housing areas within a centre's territory;

q —index of the group of enterprises in sector k $(q \in Q_k)$;

Q_k —set of possible groups of enterprises in sector k;

Q_j —set of groups of enterprises in the sector that includes enterprise j;

ψ —index of the centre's municipal engineering services $(\Psi \in \Phi)$;

Φ —set of the types of systems of the centre's municipal engineering services;

δ_ψ —index of an object of the point infrastructure ensuring functioning of the engineering services $(\delta_\psi \in \bigcup\limits_k \Delta_k)$;

Δ_k —set of objects δ_ψ in sector k;

λ —index of a section of the system of engineering services;

Λ_{δ_ψ} —set of the sections of the system abutting directly on object δ_ψ;

Λ_ψ —set of the sections of the system of engineering services ψ;

φ —index of a form of access road or railway;

$\overline{\Phi}$ —set of the sidings and access roads to sectors (roads, railway lines, etc.);

n —index of a group of areas in a sector;

N_k —set of the groups of areas of sector k differing from one another in scale of the inputs in preparing them for industrial building;

μ —index of a category of labour (μ_1—male labour; μ_2—female labour; $\mu \in \Theta$);

Θ —set of a category of manpower;

$\overline{\mu}$ —index of the use of manpower as interchangeable;

r —index of a variant of the functioning or exploitation of production facilities or resources ($r \in R_\psi \cup R_{\delta_\psi} \cup \overline{R}$; no variants of the functioning of facilities of the specialisation industries are differentiated by capacity);

R_ψ —set of the variants of the technical equipping of sectors of the system of engineering services of type ψ;

R_{δ_ψ} —set of the capacity variants of object δ_ψ;

$\overline{R}$ —set of the variants for organising a housing area.

MATRIX COEFFICIENTS

δ_j^q —output of product (type j) of the q group of enterprises per unit rate of functioning of this group;

$\Pi^{q\psi\lambda}$ —utilisation of the carrying capacity of sector λ of engineering services system ψ by the q group of enterprises per unit rate of functioning of this group;

$\overline{\Pi}_\lambda^\psi$ —utilisation of the carrying capacity of sector λ of system ψ of the engineering services per head of population;

$\Pi_k^{q\varphi}$ —utilisation of the carrying capacity of access way k of type φ of the q group of enterprises;

T_q^μ, $T_{\delta_\psi}^{\mu r}$ —demand for μ-category labour corresponding to the q group of enterprises and objects

δ_ψ for different variants r of their development $(r \in R_{\delta_\psi})$;

$T^{\bar{\mu}}_{q}$, $T^{\bar{\mu}r}_{\delta_\psi}$ —demand of the enterprises and facilities already mentioned for interchangeable labour $(r \in R_{\delta_\psi})$;

$s^{\delta_\psi r}$ —size of the area occupied by facility δ_ψ under development variant r;

s_q —total area occupied by the enterprises of group q;

$A^{\psi\lambda r}$ —increment of carrying capacity of section λ of system ψ of engineering services through implementing development variant r $(r \in \in R_\psi)$;

$A^{\delta_\psi r}$ —increment of the capacity of facility δ_ψ through implementing development variant $(r \in R_{\delta_\psi})$;

$A^{r}_{\bar{k}}$ —increment of population growth under variant r of development of housing area $\bar{k}$ $(\bar{k} \in \bar{K},\ r \in \bar{R})$;

γ demographic load factor;

γ^* coefficient of the ratio of male and female labour in the population;

ε_μ —specific weight in the total population of workers of category μ employed in the services sphere;

α_μ —specific weight in the total population of labour of category μ;

$\beta_{\mu k\bar{k}}$ —maximum specific weight of labour of category μ employable in the facilities of sector k $(k \in K)$, in the total population of housing area $\bar{k}$ $(\bar{k} \in \bar{K})$.

CONSTRAINTS

S^n_k —area of group n in sector k.

FUNCTION DESIGNATORS

$c_k^{q\psi\lambda}$ —adjusted costs of the building and functioning in sector k of the group q of enterprises using different systems of engineering services of the same type ψ, and different sectors λ of these systems;

$c^{\psi\lambda r}$ —adjusted costs of the building and functioning of sector λ of system ψ of engineering services under development variant r;

$c^{\delta_\psi r}$ —adjusted costs of the building and functioning of facility δ_ψ under variant r of the production capacity increment;

c_k^{φ} —adjusted costs of the building and functioning of the φ-type of access to sector k;

c_{kn} —adjusted costs of developing the territory of group n in sector k;

$c_{\bar{k}k}$ —adjusted costs of commuting one worker from housing area $\bar{k}$ to facilities in sector k;

$c_{\bar{k}}^{r}$ —adjusted costs of the building and functioning of housing area $\bar{k}$ under variant r for its development (not including the value of previously created assets of the municipal infrastructure);

$\bar{c}$ —input associated with the maintenance of one member of the able-bodied population who is not involved in social production.

VARIABLES

$x_k^{q\psi\lambda}$ —rate of functioning of the q group of enterprises in sector k, using different sectors λ of different system of ψ-type engineering services;

$x^{\psi\lambda r}$ —rate of utilisation of the carrying capacity of sector λ of the ψ system of engineering services under development variant r ($r \in \in R_\psi$);

$x^{\delta_\psi r}$ —rate of functioning of facility δ_ψ under variant r of the increment of production capacity ($r \in R_{\delta_\psi}$);

x_{kn} —rate of development of group n of the territory of sector k;

x^{φ}_{k} —rate of utilisation of type φ access to sector k;

$l^{\mu}_{\bar{k}k}$ —numbers of category μ manpower in housing area $\bar{k}$ employed on facilities located in sector k;

$l^{\bar{\mu}\mu}_{\bar{k}k}$ —the same for interchangeable labour of category μ;

$\bar{x}^{r}_{\bar{k}}$ —rate of utilisation of variant r of the development of housing area $\bar{k}$;

$y^{\mu_1}_{\bar{k}}$, $y^{\mu_2}_{\bar{k}}$ —numbers of men and women respectively, not employed in social production in housing area $\bar{k}$.

We write the model of the spatial structure of an industrial centre as follows:

Find the non-negative values of $x^{q\psi\lambda}_{k}$, $x^{\psi\lambda r}$, $x^{\delta_\psi r}$, x_{kn}, x^{φ}_{k}, $l^{\mu}_{\bar{k}k}$, $l^{\bar{\mu}\mu}_{\bar{k}k}$, $\bar{x}^{r}_{\bar{k}}$, $y^{\mu_1}_{\bar{k}}$, $y^{\mu_2}_{\bar{k}}$, that will satisfy the following constraints.

Each of the given enterprises of the specialisation industries must be sited within the industrial centre:

$$\sum_{k\in K}\sum_{q\in Q_j}\sum_{\psi\in\Phi}\sum_{\lambda\in\Lambda_\psi} \delta^q_j x^{q\psi\lambda}_k = 1 \left(j \in J,\ \delta^q_j = \begin{cases} 1 \text{ for } q \in Q_j \\ 0 \text{ for } q \notin Q_j \end{cases} \right). \quad (9.1)$$

The carrying capacity of all sectors of each type of system of engineering services must meet the needs of the facilities utilising that sector:

$$\sum_{k\in K}\sum_{q\in Q_k} \Pi^{q\psi\lambda} x^{q\psi\lambda}_k + \sum_{\bar{k}\in\bar{K}}\sum_{r\in\bar{R}} \bar{\Pi}^{\psi}_{\lambda} \bar{x}^{r}_{\bar{k}} - \\ - \sum_{r\in R_\psi} A^{\psi\lambda r} x^{\psi\lambda r} \leqslant 0 \quad (\lambda \in \Lambda_\psi,\ \psi \in \Phi). \quad (9.2)$$

The rate of utilisation of the individual variants of the development of the sectors of each type of system of engineering services must not exceed their nominal carrying capacity:

$$x^{\psi\lambda r} \leqslant 1 \quad (\lambda \in \Lambda_\psi,\ \varphi \in \Phi,\ r \in R_\psi). \tag{9.3}$$

Correspondence of the capacity of interconnected facilities of the point and linear infrastructure must be ensured for all types of system of engineering services:

$$\sum_{r \in R_{\delta_\psi}} A^{\delta_\psi r} x^{\delta_\psi r} - \sum_{\lambda \in \Lambda_{\delta_\psi}} \sum_{r \in R_\psi} A^{\psi\lambda r} x^{\psi\lambda r} \geqslant 0 \quad (\psi \in \Phi). \tag{9.4}$$

The rate of utilisation of the individual variants for the development of facilities of the point infrastructure will not exceed their nominal capacity:

$$x^{\delta_\psi r} \leqslant 1 \quad (\delta_\psi \in \bigcup_k \Delta_k,\ r \in R_{\delta_\psi}). \tag{9.5}$$

The carrying capacity of all types of access way abutting on the separate sectors of the sub-areal will meet the freight needs of all the facilities located in these sectors:

$$\sum_{\psi \in \Phi} \sum_{\lambda \in \Lambda_\psi} \Pi_k^{q\varphi} x_k^{q\psi\lambda} - x_k^{\varphi} \leqslant 0 \quad (k \in K,\ \varphi \in \overline{\Phi},\ q \in Q_k). \tag{9.6}$$

The area used in each sector will satisfy the needs of all the facilities located in that sector:

$$\sum_{q \in Q_k} \sum_{\psi \in \Phi} \sum_{\lambda \in \Lambda_\psi} s_q x_k^{q\psi\lambda} + \sum_{\delta_\psi \in \Delta_k} \sum_{r \in R_{\delta_\psi}} s^{\delta_\psi r} x^{\delta_\psi r} -$$
$$- \sum_{n \in N_k} x_{kn} \leqslant 0 \quad (k \in K). \tag{9.7}$$

The area employed by each group in a sector must not exceed that available:

$$x_{kn} \leqslant S_k^n \quad (k \in K,\ n \in N_k). \tag{9.8}$$

The total labour force employed by the facilities of each sector must meet the needs of all the facilities of that sector:

(i) by separate categories

$$\sum_{q \in Q_k} \sum_{\psi \in \Phi} \sum_{\lambda \in \Lambda_\psi} T_q^{\mu} x_k^{q\psi\lambda} + \sum_{\delta_\psi \in \Delta_k} \sum_{r \in R_{\delta_\psi}} T_{\delta_\psi}^{\mu r} x^{\delta_\psi r} - \sum_{\bar{k} \in \overline{K}} l_{\bar{k}k}^{\mu} \leqslant 0$$
$$(\mu \in \Theta,\ k \in K); \tag{9.9}$$

(ii) as regards interchangeability

$$\sum_{q\in Q_k}\sum_{\psi\in\Phi}\sum_{\lambda\in\Lambda_\psi} T^{\bar{\mu}}_{q} x^{q\psi\lambda}_{k} + \sum_{\delta_\psi\in\Delta_k}\sum_{r\in R_{\delta_\psi}} T^{\bar{\mu}}_{\delta_\psi} x^{\delta_\psi r} - \\ - \sum_{\bar{k}\in\bar{K}}\sum_{\mu\in\Theta} l^{\bar{\mu}\mu}_{\bar{k}k} \leqslant 0 \quad (k\in K). \tag{9.10}$$

The scale (capacity) of each housing area is determined by the number of workers settled in it:

$$\gamma \sum_{k\in K}\sum_{\mu\in\Theta} (1+\varepsilon_\mu)\,(l^{\mu}_{\bar{k}k} + l^{\bar{\mu}\mu}_{\bar{k}k}) + y^{\mu_1}_{\bar{k}} + \\ + y^{\mu_2}_{\bar{k}} - \sum_{r\in\bar{R}} A^{r}_{\bar{k}}\bar{x}^{r}_{\bar{k}} \leqslant 0 \quad (\bar{k}\in\bar{K}). \tag{9.11}$$

The rate of utilisation of the different variants for building housing areas must not exceed their nominal capacity:

$$x^{r}_{\bar{k}} \leqslant 1 \quad (\bar{k}\in\bar{K},\ r\in\bar{R}). \tag{9.12}$$

The numbers of male and female labour in all housing areas must correspond to certain proportions between them:

$$(1+\varepsilon_{\mu_1}) \sum_{k\in K} (l^{\mu_1}_{\bar{k}k} + l^{\bar{\mu}\mu_1}_{\bar{k}k}) - \gamma^{*}(1+\varepsilon_{\mu_2}) \sum_{k\in K} (l^{\mu_2}_{\bar{k}k} + l^{\bar{\mu}\mu_2}_{\bar{k}k}) - \\ - y^{\mu_1}_{\bar{k}} + y^{\mu_2}_{\bar{k}} = 0 \quad (\bar{k}\in\bar{K}). \tag{9.13}$$

Employment of the male and female labour of each housing area on remotely located jobs is limited:

$$l^{\mu}_{\bar{k}k} + l^{\bar{\mu}\mu}_{\bar{k}k} - \beta_{\mu k\bar{k}}\alpha_{\mu} \sum_{r\in\bar{R}} A^{r}_{\bar{k}}\bar{x}^{r}_{\bar{k}} \leqslant 0 \quad (\bar{k}\in\bar{K},\ k\in K,\ \mu\in\Theta). \tag{9.14}$$

The aggregate input on the building and functioning of all groups of enterprises and all created facilities of the industrial centre must be minimal:

$$\sum_{q\in Q_k}\sum_{k\in K}\sum_{\psi\in\Phi}\sum_{\lambda\in\Lambda_\psi} c^{q\psi\lambda}_{k} x^{q\psi\lambda}_{k} + \sum_{\psi\in\Phi}\sum_{\lambda\in\Lambda_\psi}\sum_{r\in R_\psi} c^{\psi\lambda r} x^{\psi\lambda r} + \\ + \sum_{\delta_\psi\in\Delta_k}\sum_{r\in R_{\delta_\psi}} c^{\delta_\psi r} x^{\delta_\psi r} + \sum_{k\in K}\sum_{\psi\in\bar{\Phi}} c^{\varphi}_{k} x^{\varphi}_{k} + \sum_{k\in K}\sum_{n\in N} c_{kn}x_{kn} + \\ + \sum_{k\in K}\sum_{\bar{k}\in\bar{K}}\sum_{\mu\in\Theta} \bar{c}_{\bar{k}k}\,(l^{\mu}_{\bar{k}k} + l^{\bar{\mu}\mu}_{\bar{k}k}) + \\ + \sum_{\bar{k}\in\bar{K}}\sum_{r\in\bar{R}} \bar{c}^{r}_{\bar{k}}\bar{x}^{r}_{\bar{k}} + \sum_{\bar{k}\in\bar{K}} \bar{c}\,(y^{\mu_1}_{\bar{k}} + y^{\mu_2}_{\bar{k}}) \to \min. \tag{9.15}$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

The effect of the factors considered in the formulation of the problem on the location of enterprises is reflected either directly in the model through the introduction of corresponding constraints or indirectly through the function designators.

The special feature of an industrial centre is the fact that the technological connections between its enterprises do not have autonomous significance because the inputs on effecting them are taken into account through the enterprises' use of elements of the production infrastructure (access ways, separate types of engineering services, and so on). The siting of technologically connected enterprises in one sector excludes any need to use several elements of the infrastructure (and in some cases to build them) and makes it possible to reduce inputs on the organisation and functioning of the centre as a whole. Only the indirect connections between enterprises arising from use of the production and social infrastructure, local resources, and labour are, therefore, considered in the model.

The location of enterprises in the different sectors of the sub-areal of an industrial centre, and the different variants of their concentration (groups) in one sector, presuppose a dissimilar degree of utilisation of the carrying capacity of the sectors of the systems of engineering services (water mains, heat mains, electricity supply lines, sewerage, etc.). That presents an opportunity of using different variants of the equipping of these sectors; the size of the inputs per unit of the resources delivered to enterprises will consequently vary. The total consumption of resources by enterprises, however, will remain unaltered since the production capacities of enterprises of the specialisation industries will remain the same for the centre under all variants of their functioning [equation (9.1)]. By virtue of that a different location of enterprises leads to a change in the value of the total input on the building and functioning of the centre. In order to allow for possible changes in the size of the inputs, two groups of constraints are introduced into the model: namely (9.2) and (9.3). The inequalities (9.2) are intended to determine the carrying capacity of thesectors of the engineering services needed for the functioning of enterprises under different schemes of their location in the centre. Taking account of the requirement of minimising inputs as a whole for the centre gives these inequalities the role of a connecting link between the location scheme of enterprises and the development of the engineering services, which are an important element of the production infrastructure. The inequalities (9.3) represent the constraints on the choice of variants of the functioning of sectors of the engineering services with different carrying capacities.

A whole number of facilities of the point infrastructure (sub-stations of the electricity system, pumping stations, sewage farms, filtering beds, etc.) are connected with the engineering services. This link finds reflection through the system of inequalities (9.4). The capacity of the facilities needed for functioning of the sectors of the services under the different variants of carrying capacity is determined by means of constraints (9.4).

The identification of facilities of the point infrastructure and the survey of different variants by capacity in the model are associated with the fact that the specific inputs on the building and functioning of a unit of capacity depend on the latter's size, and eventually on how many facilities are to be built and what the capacity of the individual facilities will be. The variant of their functioning that corresponds to their determined capacity is selected through constraint (9.5).

One of the essential elements of the production infrastructure of a centre is transport. At this stage of the investigation the access ways to the sectors of its area are treated as autonomous objects in the model. Their carrying capacity is assumed to be adequate to carry the freight traffic of all enterprises that could be built in the sector. For that purpose allowance must be made for the fact that the siting of even one enterprise in a sector calls for the building of access ways. The correlations of this type are represented by the system of inequalities (9.6).

The effect of access ways on the location scheme for enterprises in a centre makes itself felt through a saving in inputs on their construction and exploitation when a different number of sectors is employed by industry. If all the sectors are utilised the total input will be maximum; a reduction is possible by concentrating enterprises in a minimum number of sectors. It is desirable from the angle of reducing inputs on the access ways of an enterprise to concentrate on some of the centre's sectors. This, however, can have negative consequences linked with the conditions for exploiting the area of each sector. Initially enterprises will use that part of a sector's territory (industrial sub-areal) for which development costs are minimal. Then as the need for space within a given sector increases, industrial sub-areals less suitable for industry, and with higher development inputs, will be developed. This is allowed for in the model through the system of inequalities (9.7) and (9.8). Constraints (9.7) help determine enterprises' general need for the land of a sector and indicate possible ways of satisfying this need. Constraints (9.8) define the maximum possible scale of the industrial sub-areal calling for a certain input on engineering preparation.

One of the most important factors determining the siting of enterprises in a centre is labour. Its effects are various.

(1) The effect of labour comes out in the inputs on workers' commuting between a centre's housing estates and their places of work. The latter are taken as the sectors of the proposed location of groups of enterprises and facilities. The location of such groups in sectors remote from housing estates calls for large inputs on the transportation of workers, and increases the grand total of inputs on the building and functioning of the centre. There are two ways of reducing these inputs: (a) by changing the location of the enterprises and facilities mentioned in such a way that commuting to places of work will call for lower inputs; or (b) by arranging settlement in the centre in a corresponding way. This reciprocal effect of the schemes for siting enterprises and facilities and for settlement is reflected in the systems of inequalities (9.9) and (9.10).

(2) The effect of labour on the location of groups of enterprises and facilities in a centre makes itself felt through a difference in the inputs on providing the population with services in the various housing estates and under different variants of the centre's development. As a rule, both differences are manifested at the same time, which calls for con-

sideration of all possible variants of the development of housing estates when the problem of their scale is solved, which is done by means of constraints (9.11). The possibility of one variant or another for developing a housing estate being chosen is determined by the inequalities (9.12).

(3) The effect of labour is also manifested through a difference in the structure of employed and available manpower in the residential areas (neighbourhoods). These differences can arise as a consequence of the sectoral structure assigned to the centre and the specific location of groups of enterprises and facilities in it.

With a given structure of the specialisation industries it will be impossible to reduce inputs on providing services for the labour brought into the centre but not employed. Only labour of certain categories, obviously, can be fully employed. To take that into account the extended structure of labour (men and women separately) is considered in the model. The employment of labour by each category, allowing for its partial interchangeability, is considered in constraints (9.10).

With a change in the location of enterprises and facilities the inputs on providing services for the public may be altered in magnitude. This possibility is due to two factors. (1) The enterprises of the various industries employ male and female labour to different degrees. The combinations of enterprises in sectors can, therefore, be so chosen that their demand for certain categories of labour will correspond in the aggregate to the structure of the latter in the neighbouring housing areas. (2) There is a certain accessibility of enterprises for part of the population as regards travelling time. Here, too, inputs on providing services for the public can be reduced by the siting of enterprises and by the choice of appropriate combinations of them in sectors, reducing the total numbers of people by separate housing estates or in the centre as a whole.

The employment of labour at distant facilities is taken into account in the model by means of the inequalities (9.14), while correspondence of the structure of the manpower available in the composition of the population and employed in the centre's economy is taken into account by means of equations (9.13).

The model's objective function (9.15) reflects the requirement of minimising inputs on the building and functioning of the centre within the context of the enumerated constraints. By virtue of that the function includes the adjusted costs of the building and operation of groups of enterprises in different sectors, the engineering services and their associated facilities, and access ways, on developing the territory of sectors, on providing services for the manpower in the centre's housing estates, and on workers' commuting to work.

The links between the different elements of a centre s economy not reflected in the model's constraints are also reflected through the function designators. These are the links between enterprises that do not fall into one group or another, between the different systems of engineering services, between the facilities associated with these different systems, between the elements of the centre's economy enumerated above and the access ways to sectors, and between all these elements and the manpower resources, and so on. The system of links listed here is varied and complex and exerts a great influence on the location of enterprises in an industrial centre, and on the composition and developing of its production and social infrastructure.

MODEL OF THE PROCESS OF FORMING A TERRITORIAL INDUSTRIAL COMPLEX

L. I. Sevastyanov

1. FORMULATION AND MATHEMATICAL ECONOMIC MODEL OF THE PROBLEM

In addition to determining the production structure and scheme for the spatial organisation of the economy the plans for forming a territorial industrial complex also include such matters as establishing the sequence for building industrial enterprises and other facilities of all industries and deciding the sequence for developing each type of mineral deposit not considered in the models described above. These matters must be decided in close conjunction with the problem of organising the network of inhabited places (system of settlement) and developing the building industry.

The building industry performs a most important role in the development of a complex's economy. In the early stages of development it composes a large part of the population. Later, however, its role is not so decisive, since the demand on its capacities diminishes as the building programme is completed; and when the work of building the region's production infrastructure and developing the towns has been finished, its role consists (a) in maintaining the volume of production at a certain level and carrying out urban building in accordance with the rate of natural population growth and the tasks of improving public and cultural amenities and services, and (2) in carrying out major repairs and technical re-equipping of both productive and non-productive fixed assets. Thus, it becomes important, and paramount, to decide such matters as substantiation of the volume, sequence, and duration of building work, and of the development and siting of the enterprises of the local building materials industry and of prefabrication factories.

In complexes with a predominance of extractive industry a factor like the limited nature of the reserves in individual deposits has a vital effect on the development and location of the facilities of industry and of the technical and social and amenities infrastructure. In that connection matters of the spatial organisation of industry and shaping of the system of settlement must be decided, allowing for the duration of the town-forming functions of each of the units of the extractive industries.

The productive and non-productive fixed assets built in various periods of the development of a complex, and their volume and location, substantially determine the development and location of industrial and services facilities in subsequent periods. It is, therefore, also necessary to allow in the model for the development of a complex, for the temporal links of the planning objects as well as considering their spatial connections in separate periods (years). All this calls for a dynamic approach to solving matters of the forming of a complex, which must be reflected in the formulation and the mathematical economic model of the problem.

Optimisation of the forming of a complex consists in *determining a long-range plan of the development and location of the complementary and building industries, the social and services infrastructure, the economic, cultural, social and amenity functions of the industrial centres by separate stages of the complex's development that will ensure fulfilment of the output targets for products of the specialisation industries, taking into account the factors of the spatial organisation of industry and of the services sphere, with minimum aggregate inputs on the building and exploitation of all elements of the complex's economy, including its social and amenity infrastructure, while observing the conditions for the population's standard of living and the employment of labour.* In other words, the problem of this stage of intraregional territorial industrial planning is to optimise the process of the forming of a complex. In doing that much attention is paid to ensuring certain conditions as regards the population's standard of living.

One of the indicators of the population's standard of living is leisure, the significance of which is growing as the well-being of the working people rises. The degree of leisure largely decides the level of an individual's intellectual development and sometimes, too, determines his or her physical development and health.

A factor affecting the amount of leisure is the location of residential zones and places of work, and the location and organisation of the work of services and cultural facilities. The population of cities spend less time on visiting cultural and service institutions than do the population of small inhabited points, who have to journey from one village or community to another for separate types of service; workers in large communities, however, spend much time travelling to and from work, because the built-up and industrial zones of a town must, according to hygienic planning norms, be separated. The bigger a town the longer the average commuting distance (journey) to work, other things being equal, and the longer the average time taken.

Large communities built in regions of development of the extractive industries may serve a larger number of mineral deposits than smaller ones. The average distance to the deposits is, therefore, longer from large communities than from small ones. (Another relationship between the size of a community and the length of the journey to work is not excluded.) A similar position exists in areas of the lumbering industry and certain other industries in which the objects of the application of labour are fixed in location (e.g. agriculture). From this aspect workers living in small communities have certain advantages, and spend less time travelling to work than the inhabitants of bigger ones.

The time spent on journeying to work and for cultural and welfare services reduces leisure and diminishes a person's opportunities for efficient recreation and intellectual development. Attention has, therefore, to be paid to this expenditure of the public's time when matters of settlement are being decided. It can be reflected in the mathematical and economic model by analogy with other components of the standard of well-being when the constraints for types of service and manpower are being established, or when the coefficients by which each of the modes of settlement is described are being calculated. But if we assume that the main components of the standard of well-being (housing conditions, public utilities, shops, medical, social, and cultural services, public catering, the character of work, opportunities for choosing and changing jobs, and for spending leisure time) are the same in the different variants of settlement, and that these variants differ from each other solely in amount of leisure (which is determined by the extent of the expenditure of time on journeys to work and for cultural and public services), then the following statement will probably be correct: the best variant of settlement is that under which the public spends the least time on commuting.

The level of provision of housing and cultural and municipal services for the population of a certain area, and consequently their opportunities of employing their leisure, are determined at one of the stages of optimal territorial industrial planning that precedes solution of the problems of organising and developing a complex, i.e. these indicators can be considered as given. The indices describing the character of work and opportunities of choosing and changing jobs can also be taken as given, after the optimal problems of systems of higher rank have been solved. The differences between variants of settlement will thus consist solely in the different extent of the public's expenditure of time on commuting (to work and for services), i.e. may be realised in the given conditions of optimising the organisation of a complex, not only in minimum aggregate inputs on the building and exploitation of the set of facilities of industry and technical and social services infrastructure but also in a minimum expenditure of the public's time on commuting.

It is assumed that the output volume of the specialisation industries (in a break-down by separate enterprises) and the limits on deliveries of various types of resource from outside will be known before the problem is tackled. The following matters are solved for each period of the development of a complex when the process of forming it is being optimised:

(a) the direction and rate of freight traffic and other connections between the complex's production facilities;

(b) the structure, scale of development, and economic, cultural, and social and welfare functions of the complex's industrial centres;

(c) the direction and intensity of cultural, social, and welfare links between industrial centres;

(d) the forms of developing new areas and the regimes of work and rest in production facilities located at a considerable distance from communities;

(e) the distribution, reproduction, employment, and migration of labour and the location of population by centres of the complex;

(f) the distribution of freight traffic due to complementary industries among the different types of transport, the use of the transport system, and determination of the paths of development of its constituent sectors.

When these problems are being solved the following must be taken into account: the technological links between enterprises and production activities and the effect of their mutual location within the complex; the volume and proportions of the development and location of each of the planning objects existing at the beginning of each of the periods considered; the length of the production cycle of the building of new industrial and services facilities; the possibility of employing one and the same local and outside resources for the building, functioning, and further development of production units of the various branches of the production and social infrastructure; the conditions of the structure, reproduction, employment, and migration of labour and population; the carrying capacity of the transport system's sections and its opportunities for growth under different variants of development.

The effect of these factors is taken into account during the modelling of the forming of a complex either directly, by introducing appropriate equations and inequalities, or indirectly by including the inputs caused by them in the function designators.

In order to allow for the features of the functioning, location, and effect of all units of the complex's economy on the organisation of communities, these units are divided into two groups:

(1) enterprises, institutions, and organisations of town-forming significance (town-forming objects), and

(2) facilities of the urban social and municipal infrastructure, which includes enterprises and institutions providing cultural and social amenities; administrative institutions, and so on, serving the given inhabited point. Town-forming objects are all industrial, power, and agricultural enterprises and warehouses (with the exception of these serving only the public of a given settlement), enterprises,

institutions, and other facilities of outside transport (rail, marine, river, air, and road transport, and pipelines), building and research and development institutions, higher and specialised educational establishments, health resort institutions (resorts and convalescent homes, holiday centres, etc.), administrative, social, and economic institutions of extra-city or extra-community significance, facilities of intercommunity cultural and public services that town-planning standards and rules require to be located in one of the group of settlements, and whose composition and capacity are calculated so as to cater for the inhabitants of the whole group of communities. The workers and employees of town-forming objects belong to the group of town-forming personnel, while the workers of the local urban social and amenity infrastructure belong to the group of service personnel.

This grouping of a complex's enterprises, institutions, and organisations, and of their staffs, ensures unification of the objects of planning in the mathematical economic model of the organising of a complex that we present below. The following notation is used in the model.

INDICES AND SETS

I —set of sub-areals suitable for locating communities and town-forming objects $(j \in J_1)$, including sub-areals lying outside the economic region being considered;

i, l —elements of set I $(i \in I, l \in I)$;

I_1 —sub-sets of sub-areals (communities) located within the region under consideration $(I_1 \subset \subset I)$;

J —set of the town-forming objects (enterprises, organisations, institutions, etc.), including facilities of the social and amenity infrastructure of intercommunity significance and sections of the transport system;

j, f —indices of town-forming objects $(j \in J, f \in \in J)$;

J_I —sub-set of objects j, f located within communities $(J_I \subset J)$;

J_i, J_l —sub-set of town-forming objects either located in communities i, l or attracted to them by transport lines and consumption of natural resources $(J_i \subset J,\ J_l \subset J)$;

J_n —sub-set of town-forming objects that are sections of the transport system $(J_n \subset J)$;

J_q —sub-set of objects j offering products (services) of type q $(J_q \subset J)$;

Q —set of the types of product and service under consideration;

q —indices of the separate types of product or service $(q \in Q)$;

Q_1 —sub-set of non-transportable types of product or service $(Q_1 \subset Q)$;

Q_2 —sub-set of types of product for which output targets have been fixed $(Q_2 \subset Q)$;

Q_3 —sub-set of types of product of building organisations $(Q_3 \subset Q)$;

Q_4 —sub-set of the types of product and service of facilities of the social and amenity infrastructure of intercommunity significance $(Q_4 \subset Q)$;

R_j —set of development variants of object j;

r —indices of development variants of objects $(r \in R_j)$;

S —set of types of natural resource (land, water, atmosphere, etc.);

s —indices of the various types of natural resource $(s \in S)$;

T —set of the indices of the periods under review in the region's development $(T = \{1, 2, \ldots \ldots \tau\})$;

t, $t + \varphi$ —indices of the elements of set T $(t \in T;\ \{t + \varphi\} \in T)$;

μ —categories of labour by sex $(\mu \in \mathrm{M},\ \mathrm{M} = \{1, 2\};\ \mu = 1$—men, $\mu = 2$—women);

P —set of limited resources;

ρ —indices of separate types of limited resource $(\rho \in \mathrm{P})$;

Ψ —set of routes possible on the region's transport system;

ψ —indices of the elements of set Ψ $(\psi \in \Psi)$;

Ψ_j —sub-set of routes running over sector j $(\Psi_j \subset \Psi)$;

Ψ_q —sub-set of routes for shipping out product q $(\Psi_q \subset \Psi)$.

MATRIX COEFFICIENTS

a_j^{rt}—labour productivity of worker of section j of the transport system developed in period t under variant r, in the indices of carrying capacity;

a_{qj}^{rt}—output per worker of product (service) of type q by object j, developed in period t under variant r;

a_q^{τ}—average production per worker engaged in type q building work, as planned for the last of the periods under consideration;

b_q—coefficient of the conversion of a unit of product (type q) into the index of the volume of freight;

b_{ql}^{t}, $b_{ql}^{t+\varphi}$—quantity of q-type product or service needed in period t, $t+\varphi$ for the complex of objects of a town's social and amenity infrastructure, per worker of community l;

b_{qf}^{rt}, $b_{qf}^{r,t+\varphi}$—quantity of q-type product or service per worker needed in period t, $t+\varphi$ for object f being developed by variant r;

b_q^{τ}—average volume of q-type building work per worker planned for period τ;

b_{sj}^{rt}—demand per worker for natural resource s for the functioning of object j in period t by variant r;

b_s^t—demand per worker for natural resource s for the functioning of facilities of the town's social and amenity infrastructure in period t;

$b_{\rho j}^{rt}$—quantity of ρ-type limited resource needed per worker for object j being developed by variant r in period t;

$g_{f\psi}^{rt}$—volume of raw materials, supplies, fuel, and other freights along route ψ, needed per worker for the functioning of production facility f being developed by variant r in period t;

p_j^{rt}—specific weight of interchangeable male and female labour in the total demand for labour of facility j being developed by variant r in period t;

$p_{\mu j}^{rt}$—specific weight of μ-category labour in the total labour needs of facility j, being developed by variant r in period t;

α_q^t—coefficient of the possible use of q-type product in period t produced in the preceding period;

α^t—specific weight of town-forming people in the total population in period t;

β_{ql}^t—quantity of q-type product or service needed per worker of community l for major repairs and upkeep of the set of facilities of the town's social and amenity infrastructure at the standard of services envisaged for period t;

β_{qj}^{rt}—quantity of q-type product or service needed per worker for major repairs and upkeep of facility f, being developed by variant r at the standard of technical equipping of similar facilities in period t;

$\gamma_{i_1}^{t}$—number of working males (per arriving woman worker) arriving in the families of women in period t drawn to community i to meet a shortage of female labour;

$\gamma_{i_2}^{t}$—number of working females (per arriving male worker) arriving in community i in period t as a second member of families attracted to the community to meet a shortage of male labour;

ε_{μ}^{t}—growth factor in period t of manpower of μ category compared with the preceding period (allowing for natural reproduction);

$\theta_i^{t,\min}$ $\theta_i^{t,\max}$—coefficients defining the minimum and maximum ratios of male and female labour in community i in period t;

$\lambda_{i_1}^{i}$—number of working males leaving community i in period t together with female workers (per working women leaving the community due to a fall in the demand for female labour);

$\lambda_{i_2}^{t}$—number of working women leaving community i in period t as second members of families (per male worker leaving the community due to a fall in the demand for male labour);

$\nu_{ql}^{t(t)}$, $\nu_{ql}^{t(t+\varphi)}$ —the part of the demand for q-type product or service that must be produced in period t to ensure commissioning of the set of facilities of the town social and amenity infrastructure of community l in period t, $t+\varphi$;

$\nu_{qf}^{rt(t)}$, $\nu_{qf}^{rt(t+\varphi)}$—the part of the demand for q-type product or service that must be produced in period t so as to commission facility j under variant r in period $t, t+\varphi$;

π_{μ}^{t}—specific weight of μ-category labour in the total labour needs of the facilities of the town social and amenity infrastructure in period t;

π^{t}—specific weight of interchangeable male and female labour in the total needs of the facilities of the town social and amenity infrastructure in period t;

ω^{t}—specific weight of the service personnel employed in the facilities of the town social and amenity infrastructure in the total population in period t.

CONSTRAINTS

A_{jq}^{t}—planned volume of output of q-type product by facility j in period t;

A_{q}^{t}—planned output target for q-type product in period t;

B_{si}^{t}—maximum consumption of natural resource s in subareal i in period t;

B_{ρ}^{t}—maximum consumption of limited resource ρ in period t;

F_{j}^{rt}—minimum number of workers of object j for its functioning in period t by variant r;

G_{j}^{t}—transit freight traffic on section j of the transport system in period t;

P_{ij}^{r0}—number of workers of community i employed at the beginning of the plan period at facility j being developed by variant r;

$P_{i\mu}^{0}$—number of μ-category workers, settled in community i at the beginning of the plan period, employed in production activities that preclude substitution of male and female labour;

$\Lambda^0_{i\mu}$—amount of μ-category labour not utilised in community i before the beginning of the plan period;

$\Pi^0_{i\mu}$—number of workers of category μ, settled in community i at the beginning of the plan period, employed in production activities permitting interchange of male and female labour;

Φ^{rt}_j—maximum number of workers of facility j under its functioning by variant r in period t;

Ω^t—maximum import of labour to the complex in period t.

FUNCTION DESIGNATORS

c^{rt}_{ij}—discounted time-value of inputs on exploiting facility j (being developed by variant r) in period t (per employee of this facility settled in community i);

c^t_i—discounted time-value of inputs on exploiting the set of facilities in period t of the town social and amenity infrastructure of community i per working person;

$h^t_{i\mu}$—discounted time-value of inputs per worker associated with maintaining non-employed labour of category μ in period t in community i (i.e. penalty for non-use of labour);

h^t_i—discounted time-value of inputs per employee associated with the maintenance in period t of unused fixed assets of the previously built set of facilities of the town social and amenity infrastructure of community i;

k^{rt}_{ij}—discounted time-value of inputs per worker employed in facility j, living in community i, needed to create this facility in period t by variant r;

k^t_i—discounted time-value of inputs needed to create in period t the set of facilities of the town social and amenity infrastructure of community i per worker.

δ_{ij}^{rt}—discounted time-value of the capital investment per worker in community i needed to bring facility j, being developed by variant r, up to the standard of technical equipping of similar facilities in period t;

δ_i^t—discounted time-value of the capital investment per worker needed to operate the facilities of the town social and amenity infrastructure of community i at the standard of services envisaged for period t;

$\varkappa_{i\mu}^t$ —discounted time-value of inputs on attracting one worker of category μ to community i in period t (fares, moving and daily travelling allowances, etc.).

$\varkappa_{qjf\psi}^t$—discounted time-value of inputs on transporting a unit of q-type product or service in period t from facility j to facility f along route ψ;

$\varkappa_{qjl\psi}^t$—discounted time-value of inputs on transporting a unit of product or service (q-type) in period t from facility j to community l by route ψ;

$\varkappa_{qj\psi}^t$ —discounted time-value of inputs on shipping a unit of q-type product produced by facility j out of the complex by route ψ in period t;

$\tau_{qjl\psi}^t$—time spent per worker by the residents of community l on obtaining a unit of q-type service rendered by a cultural or service facility of intercommunity significance j, using route ψ;

τ_{ij}^{rt}—time spent per worker of community i, employed at facility j functioning by variant r, on commuting to work in period t.

UNKNOWNS

$u_{i\mu}^t$, $u_{l\mu}^t$—labour of category μ not employed in communities i and l in period t;

v_{ij}^{rt}, v_{if}^{rt}—reduction in period t on the preceding period of the number of workers of community i employed at facilities j and f, being developed by variant r;

$v^t_{i\mu}$—amount of labour of category μ leaving community i in period t;

v^t_i, v^t_l—variables expressing the volume of the previously built town social and amenity infrastructure in communities i and l not utilised in period t;

w^{rt}_{ij}, w^{rt}_{if}—increment in period t on the preceding period in the number of workers of community i employed at facilities j and f being developed by variant r;

$w^t_{i\mu}$—amount of labour of category μ attracted to community i in period t;

w^t_i, w^t_l—number of workers of communities i and l not served at the beginning of period t by already existing facilities of the town social and amenity infrastructure;

x^{rt}_{ij}, x^{rt}_{if}—number of workers of community i employed in period t at facility j and f being developed by variant r;

$x^t_{i\mu}$, $x^t_{l\mu}$—number of workers of category μ settled in communities i and l employed in period t in industries in which male and female labour is not interchangeable;

$y^t_{i\mu}$, $y^t_{l\mu}$—number of workers of category μ settled in communities i and l employed in period t in industries in which male and female labour is interchangeable;

$z^t_{qjf\psi}$—quantity of q-type product or service produced by facility j and conveyed by route ψ for the needs of facility f in period t;

$z^t_{qjl\psi}$—quantity of q-type product or service produced by facility j and conveyed by route ψ for the needs of the town-forming objects and the facilities of the town social and amenity infrastructure of community l in period t;

$z^t_{qj\psi}$—quantity of q-type product produced by facility j for shipment out of the complex in period t by route ψ.

The mathematical economic model of the problem of optimising the forming of a complex is written as follows in the notation adopted:

Find

$$u^t_{i\mu}\geqslant 0,\ v^{rt}_{ij}\geqslant 0,\ v^t_{i\mu}\geqslant 0,\ v^t_i\geqslant 0,\ w^{rt}_{ij}\geqslant 0,\ w^t_{i\mu}\geqslant 0,\ w^t_i\geqslant 0,$$
$$x^{rt}_{ij}\geqslant 0,\ x^t_{i\mu}\geqslant 0,\ y^t_{i\mu}\geqslant 0,\ z^t_{qjf\psi}\geqslant 0,\ z^t_{qjl\psi}\geqslant 0,\ z^t_{qj\psi}\geqslant 0$$

that will satisfy the constraints listed below.

The production and distribution of all types of products and services other than transport is balanced in each period:

$$\sum_{i\in I}\sum_{r\in R_j} a^{rt}_{qj}x^{rt}_{ij}-\sum_{f\in J\setminus J_I}\sum_{\psi\in\Psi_q} z^t_{qjf\psi}-$$
$$-\sum_{l\in I}\sum_{\psi\in\Psi_q} z^t_{qjl\psi}-\sum_{f\in J\setminus J_I}\sum_{\psi\in\Psi_q}\alpha^{t+1}_q z^{t+1}_{qjf\psi}-$$
$$-\sum_{l\in I}\sum_{\psi\in\Psi_q}\alpha^{t+1}_q z^{t+1}_{qjl\psi}-\sum_{\psi\in\Psi_q} z^t_{qj\psi}\geqslant 0\ \ (j\in J_q,\ q\in Q_1;\ t\in T);\quad(10.1)$$

$$\sum_{j\in J_q}\sum_{\psi\in\Psi_q} z^t_{qjf\psi}+\sum_{j\in J_q}\sum_{\psi\in\Psi_q}\alpha^t_q z^t_{qjf\psi}-\sum_{i\in I}\sum_{r\in R_j} b^{rt}_{qf}x^{rt}_{if}-$$
$$-\sum_{i\in I}\sum_{r\in R_j}\nu^{rt(t)}_{qf}b^{rt}_{qf}w^{rt}_{if}-\sum_{i\in I}\sum_{r\in R_j}\nu^{rt(t+\varphi)}_{qf}b^{rt,\ t+\varphi}_{qf}w^{r,\ t+\varphi}_{if}-$$
$$-\sum_{i\in I}\sum_{r\in R_j}\beta^{rt}_{qf}x^{r,\ t-1}_{if}+\sum_{i\in I}\sum_{r\in R_j}\beta^{rt}_{qf}v^{rt}_{if}\geqslant 0$$
$$(q\in Q_1;\ f\notin J_I,\ t\in T);\quad(10.2)$$

$$\sum_{j\in J_q}\sum_{\psi\in\Psi_q} z^t_{qjl\psi}+\sum_{j\in J_q}\sum_{\psi\in\Psi_q}\alpha^t_q z^t_{qjl\psi}-\sum_{i\in I}\sum_{f\in J_l}\sum_{r\in R_j} b^{rt}_{qf}x^{rt}_{if}-$$
$$-\sum_{i\in I}\sum_{f\in J_l}\sum_{r\in R_j}\nu^{rt(t)}_{qf}b^{rt}_{qf}w^{rt}_{if}-\sum_{i\in I}\sum_{f\in J_l}\sum_{r\in R_j}\nu^{rt(t+\varphi)}_{qf}b^{r,\ t+\varphi}_{qf}w^{r,\ t+\varphi}_{if}-$$
$$-\sum_{i\in I}\sum_{f\in J_l}\sum_{r\in R_j}\beta^{rt}_{qf}x^{r,\ t-1}_{if}+\sum_{i\in I}\sum_{f\in J_l}\sum_{r\in R_j}\beta^{rt}_{qf}v^{rt}_{if}-$$
$$-\sum_{\mu\in M} b^t_{ql}x^t_{l\mu}-\sum_{\mu\in M} b^t_{ql}y^t_{l\mu}-\sum_{\mu\in M} b^t_{ql}u^t_{l\mu}-\nu^{t(t)}_{ql}b^t_{ql}w^t_l-$$
$$-\nu^{t(t+\varphi)}_{ql}b^{t+\varphi}_{ql}w^{t+\varphi}_l-\sum_{\mu\in M}\beta^t_{ql}x^{t-1}_{l\mu}-\sum_{\mu\in M}\beta^t_{ql}y^{t-1}_{l\mu}-$$
$$-\sum_{\mu\in M}\beta^t_{ql}u^{t-1}_{l\mu}-\beta^t_{ql}v^{t-1}_l\geqslant 0\quad(q\in Q_1,\ l\in I,\ t\in T).\quad(10.3)$$

Targets are fixed for the output of separate types of product for facilities of the specialisation industries in each period:

$$\sum_{i\in I}\sum_{r\in R_j} a_{qj}^{rt}x_{ij}^{rt}\geqslant A_{qj}^{t} \quad (j\in J_q,\ q\in Q_2,\ t\in T). \qquad (10.4)$$

The carrying capacity of each section of the transport system will ensure carriage both of freights generated by the complex's enterprises and of transit traffic:

$$\sum_{i\in I} a_j^{rt}x_{ij}^{rt}-\sum_{\psi\in\Psi_j}\sum_{i\in I}\sum_{f\in J}\sum_{r\in R_j} g_{f\psi}^{rt}x_{if}^{rt}-$$

$$-\sum_{\psi\in\Psi_j}\sum_{q\in Q_1}\sum_{j\in J_q} b_q z_{qj\psi}^{t}\geqslant G_j^{tr} \quad (j\in J_n,\ r\in R_j,\ t\in T). \qquad (10.5)$$

The number of workers of each community employed at the various town-forming objects is made up in any period of the workers employed at them in the preceding period plus the increment (or minus the decline) due to a change in volume of production or productivity:

$$x_{ij}^{rt}-x_{ij}^{r,\,t-1}-w_{ij}^{rt}+v_{ij}^{rt}=0 \quad (i\in I,\ j\in J,\ r\in R_j,\ t\in T). \quad (10.6)$$

The total number of men and women employed in social production (on town-forming objects and in the services sphere) for each community is determined separately in all periods by the processes of production and management that preclude or permit interchangeability of male and female labour:

$$\sum_{j\in J}\sum_{r\in R_j}\left(\frac{\omega^t}{\alpha^t}\pi_\mu^t+p_{\mu j}^{rt}\right)x_{ij}^{rt}-x_{i\mu}^{t}=0 \ (i\in I,\ \mu\in M,\ t\in T); \ (10.7)$$

$$\sum_{j\in J}\sum_{r\in R_j}\left(\frac{\omega^t}{\alpha^t}\pi^t+p_j^{rt}\right)x_{ij}^{rt}-\sum_{\mu\in M} y_{j\mu}^{t}=0 \quad (i\in I,\ t\in T). \qquad (10.8)$$

In all the periods considered the reproduction, employment, and migration of the male and female labour of each community are balanced:

$$x_{i\mu}^t+y_{i\mu}^t+u_{i\mu}^t-\varepsilon_\mu^t x_{i\mu}^{t-1}-\varepsilon_\mu^t y_{i\mu}^{t-1}-\varepsilon_\mu^t u_{i\mu}^{t-1}-w_{i\mu}^t+v_{i\mu}^t=0$$

$$(i\in I,\ \mu\in M,\ t\in T). \qquad (10.9)$$

For that purpose the effect on the sex structure of the labour force of the ratio of men and women among those coming to a community and those leaving is taken into account in all the periods considered:

$$\gamma_{i2}^{t} w_{i1}^{t} - w_{i2}^{t} \leqslant 0 \quad (i \in I_1,\ t \in T); \tag{10.10}$$

$$\lambda_{i2}^{t} v_{i1}^{t} - v_{i2}^{t} \leqslant 0 \quad (i \in I_1,\ t \in T); \tag{10.11}$$

$$v_{i1}^{t} - \lambda_{i1}^{t} v_{i2}^{t} \geqslant 0 \quad (i \in I_1,\ t \in T); \tag{10.12}$$

$$w_{i1}^{t} - \gamma_{i1}^{t} w_{i2}^{t} \geqslant 0 \quad (i \in I_1,\ t \in T). \tag{10.13}$$

The quantity of male and female labour in each of the complex's communities must correspond in all periods to certain proportions between them:

$$x_{i1}^{t} + y_{i1}^{t} + u_{i1}^{t} - \theta_i^{t,\,\min} x_{i2}^{t} - \theta_i^{t,\,\min} y_{i2}^{t} - \theta_i^{t,\,\min} u_{i2}^{t} \geqslant 0 \quad (i \in I_1,\ t \in T); \tag{10.14}$$

$$x_{i1}^{t} + y_{i1}^{t} + u_{i1}^{t} - \theta_i^{t,\,\max} x_{i2}^{t} - \theta_i^{t,\,\max} y_{i2}^{t} - \theta_i^{t,\,\max} u_{i2}^{t} \leqslant 0 \quad (i \in I_1,\ t \in T). \tag{10.15}$$

The volume of work on building the town social and amenity infrastructure is defined for each community in all the periods considered, or the unutilised part is revealed:

$$\sum_{\mu \in M} w_{i\mu}^{t} + \sum_{\mu \in M} (\varepsilon_{\mu}^{t} - 1)\, x_{i\mu}^{t-1} + \sum_{\mu \in M} (\varepsilon_{\mu}^{t} - 1)\, y_{i\mu}^{t-1} + \sum_{\mu \in M} (\varepsilon_{\mu}^{t} - 1)\, u_{i\mu}^{t-1} - \sum_{\mu \in M} v_{i\mu}^{t} - v_i^{t-1} - w_i^{t} + v_i^{t} = 0 \quad (i \in I,\ t \in T). \tag{10.16}$$

The total capacity of the complex's building organisations in the last of the periods considered is fixed at a level adequate to ensure extension and modernisation of production, meet population growth through natural reproduction, improve cultural and social services and amenities, and carry out major repairs of fixed productive and non-productive assets:

$$\sum_{i \in I} \sum_{j \in J_q} \sum_{r \in R_i} a_q^{\tau} x_{ij}^{r\tau} - \sum_{l \in I} \sum_{\mu \in M} b_q^{\tau} x_{l\mu}^{\tau} - \sum_{l \in I} \sum_{\mu \in M} b_q^{\tau} y_{l\mu}^{\tau} \geqslant 0 \quad (q \in Q_3). \tag{10.17}$$

The limited nature of its natural resources is taken into account in each period for each sub-areal:

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R_j} b^{rt}_{sj} x^{rt}_{ij} + \sum_{\mu\in M} b^t_s x^t_{i\mu} + \sum_{\mu\in M} b^t_s y^t_{i\mu} + \sum_{\mu\in M} b^t_s u^t_{i\mu} \leqslant B^t_{si}$$

$$(s\in S,\ i\in I;\ t\in T). \qquad (10.18)$$

The exploitation of limited resources in each period must not exceed the limits set:

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R_j} b^{rt}_{\rho j} x^{rt}_{ij} \leqslant B^t_\rho \quad (\rho\in P,\ t\in T). \qquad (10.19)$$

The applicability of some variants of the development of town-forming objects is limited by certain minimum and maximum dimensions, i.e.

$$\sum_{i\in I} x^{rt}_{ij} \in \{0,\ [F^{rt}_j,\ \Phi^{rt}_j]\} \quad (j\in J,\ r\in R_j,\ t\in T). \qquad (10.20)$$

The scale of the permitted influx of labour into the complex in each period is taken into account where necessary:

$$\sum_{i\in I_1}\sum_{\mu\in M} w^t_{i\mu} \leqslant \Omega^t \quad (t\in T). \qquad (10.21)$$

The number of workers in existing town-forming objects, and the total labour available at the beginning of the first period considered, are also taken into account for communities (sub-areals):

$$x^{r0}_{ij} = P^{r0}_{ij} \quad (i\in I,\ j\in J,\ r\in R_j); \qquad (10.22)$$

$$x^0_{i\mu} = P^0_{i\mu};\quad y^0_{i\mu} = \Pi^0_{i\mu};\quad u^0_{i\mu} = \Lambda^0_{i\mu} \quad (i\in I,\ \mu\in M). \qquad (10.23)$$

The optimisation of developing an industrial complex is broken down in turn into two criteria: (i) into the minimum aggregate discounted time-values of the inputs on building and exploiting the set of town-forming objects and of the town social amenity infrastructure:

$$L_1 = \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{r\in R_j} (c^{rt}_{ij} + \delta^{rt}_{ij})\, x^{rt}_{ij} +$$

$$+ \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{r\in R_j} (k^{rt}_{ij} - \delta^{rt}_{ij})\, w^{rt}_{ij} +$$

$$+ \sum_{t\in T}\sum_{i\in I}\sum_{\mu\in M} \left(c^t_i + \frac{1}{\varepsilon^t_\mu}\,\delta^t_i\right) x^t_{i\mu} + \sum_{t\in T}\sum_{i\in I}\sum_{\mu\in M} \left(c^t_i + \frac{1}{\varepsilon^t_\mu}\,\delta^t_i\right) y^t_{i\mu} +$$

$$+\sum_{t\in T}\sum_{i\in I}\sum_{\mu\in M}\left(h_{i\mu}^{t}+c_{i}^{t}+\frac{1}{\varepsilon_{\mu}^{t}}\delta_{i}^{t}\right)u_{i\mu}^{t}+$$

$$+\sum_{t\in T}\sum_{i\in I}\sum_{\mu\in M}\left(\varkappa_{i\mu}^{t}-\frac{1}{\varepsilon_{\mu}^{t}}\delta_{i}^{t}\right)\omega_{i\mu}^{t}+$$

$$+\sum_{t\in T}\sum_{i\in I}\sum_{\mu\in M}\frac{1}{\varepsilon_{\mu}^{t}}\delta_{i}^{t}v_{i\mu}^{t}+\sum_{t\in T}\sum_{i\in I}k_{i}^{t}w_{i}^{t}+$$

$$+\sum_{t\in T}\sum_{i\in I}h_{i}^{t}v_{i}^{t}+\sum_{t\in T}\sum_{q\in Q_1}\sum_{i\in I_q}\sum_{f\in J\setminus J_J}\sum_{\psi\in\Psi_q}\varkappa_{qjf\psi}^{t}z_{qjf\psi}^{t}+$$

$$+\sum_{t\in T}\sum_{q\in Q_1}\sum_{j\in J_q}\sum_{l\in I}\sum_{\psi\in\Psi_q}\varkappa_{qjl\psi}^{t}z_{qjl\psi}^{t}+$$

$$+\sum_{t\in T}\sum_{q\in Q_1}\sum_{j\in J_q}\sum_{\psi\in\Psi_q}\varkappa_{qj\psi}^{t}z_{qj\psi}^{t} \qquad (10.24)$$

and (ii) into the minimum expenditure of time by the population's commuting connected with satisfying their need for services of the appropriate facilities of intercommunity significance:

$$L_2=\sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\sum_{r\in R_j}\tau_{ij}^{rt}x_{ij}^{rt}+\sum_{t\in T}\sum_{q\in Q_4}\sum_{j\in J_q}\sum_{l\in I}\sum_{\psi\in\Psi_q}\tau_{qjl\psi}^{t}z_{qjl\psi}^{t}. \qquad (10.25)$$

The optimum plans obtained from criteria (10.24) and (10.25) are co-ordinated by using one of the methods of solving linear programming problems with several objective functions. A compromise plan X^0 is obtained as a result of solving the linear programming problem, for which the following notations are used:

$\overline{X}_1$ and $\overline{X}_2$—the optimum vectors of the plans corresponding to the first and second criteria;

d_1 and d_2—unknown variables expressing the rate of using the optimum plans according, respectively, to the first and second criterion, in the compromise plan;

$L_1\overline{X}_1$—value of the objective function (10.24) obtained by solving the problem for the first criterion;

$L_1\overline{X}_2$—value of the objective function (10.24) corresponding to the optimum plan by the second criterion:

$L_2\overline{X}_2$—magnitude of the objective function (10.25) when the problem is solved by the second criterion;

$L_2\overline{X}_1$—magnitude of the objective function (10.25) corresponding to the optimum plan obtained by solving the problem by the first criterion;

η —relative magnitude of the maximum deviation in respect of both criteria when the compromise plan X^0 is used.

The problem consists in finding values of $d_1 \geqslant 0$, $d_2 > 0$, and $\eta \geqslant 0$, such as will ensure a minimum deviation in the compromise plan X^0 from the optimum plans obtained by solving the problem by the first and second criteria, i.e. to obtain the minimum linear form

$$L^0 = \eta \tag{10.26}$$

with the constraints

$$\eta + \frac{L_1\overline{X}_1 - L_1\overline{X}_2}{L_1\overline{X}_1} d_2 \geqslant 0; \tag{10.27}$$

$$\eta + \frac{L_2\overline{X}_2 - L_2\overline{X}_1}{L_2\overline{X}_2} d_1 \geqslant 0; \tag{10.28}$$

$$d_1 + d_2 = 1. \tag{10.29}$$

The compromise plan is obtained as

$$X^0 = d_1\overline{X}_1 + d_2\overline{X}_2. \tag{10.30}$$

2. THE ECONOMIC SENSE OF THE MODEL'S CONSTRAINTS

Integrated development of the economy of a territorial industrial complex must ensure satisfaction of the needs of the town-forming objects and population of each town or community for various kinds of product or service. This is ensured in the mathematical model by input-output matrices of the production and distribution of goods and services [(10.1)-(10.3)]. Depending on the specific conditions of the complex's organisation these balances can envisage utilisation of the capacities of appropriate enterprises, organisations, and institutions located both within the territory under review and outside it, which makes it possible to find the most efficient way of locating and developing the production of goods and services, taking intraregional and interregional connections into account.

It is assumed [in (10.2) and (10.3)] that work on major repairs, technical improvement of fixed assets, and improving services for the public is done during each period reviewed, but that it is possible to transfer their fulfilment (in whole or in part) to preceding periods, which can be expressed in the model by the coefficients $\nu_{qf}^{rt(t)}$, $\nu_{qf}^{rt\,(t+\varphi)}$, $\nu_{ql}^{t(t)}$, and $\nu_{ql}^{t\,(t+\varphi)}$ as is done in regard to providing fixed assets for the increment in numbers of workers and population. These coefficients reflect the specific nature of the making of products with a long production cycle, e.g. the building and assembly work. When the demand of most industries for goods is being determined, therefore, the values of the coefficients are taken as zero, i.e. the scale of demand for goods and services of the current operation of facilities is determined at the established consumption norms solely b_{qf}^{rt} and a_q^{t+1} by the rate of the variables x_{if}^{rt}, $x_{l\mu}^{t}$, $y_{l\mu}^{t}$, and $u_{l\mu}^{t}$.

The coefficients α_q^t and α_q^{t+1} are also taken as zero with variables that express volumes of freights for those types of product that cannot be produced and stored until the demand arises.

Constraints (10.4), which reflect fulfilment of the assignments for production of separate types of product by each of the facilities of the specialisation industries in each period, may take the form given below in certain cases, in particular when the development of separate complexes is modelled outside the group of mathematical economic models considered in this monograph, when matters associated with determining the optimum parameters and spatial structure are resolved simultaneously with optimisation of the process of forming a complex:

$$\sum_{i\in I}\sum_{j\in J_q}\sum_{r\in R_j} a_{qj}^{rt}x_{ij}^{rt} \geqslant A_q^t\,(q\in Q_2,\ t\in T),$$

i.e. the output targets for the separate types of product of the specialisation industries are set for the complex as a whole in each period.

The sections of the transport system are treated in the proposed model as town-forming objects $j \in J_n \, (J_n \subset J)$, whose volumes of production are measured by the indices of carrying capacity [see constraint (10.5), which provides for development of the complex's transport system under the impact of the freight traffic generated by its enterprises plus transit freights].

Analysis of the process of forming a complex calls for consideration of the temporal connections of the objects of planning as well as of their spatial connections. For town-forming objects this link is ensured by a system of balanced equations (10.6), each of which provides for taking account of changes in the number of workers at a facility through the effect of the dynamics of output and productivity. Constraint (10.6) also serves as the basis for a dynamic approach to the generation of inputs on the building, functioning, and development of town-forming objects and to defining the volume of building and assembly work in each of the periods [see constraints (10.2) and (10.3)].

All the processes of production and management in town-forming objects and facilities of the municipal infrastructure can be arbitrarily divided into three groups. The first two preclude interchangeability of male and female labour, i.e. employ the labour either of men or of women only. The third group is formed by processes in which male and female labour may be interchanged. An important condition of the organisation of a production complex is that of ensuring correspondence between the demand for male and female labour and the availability of manpower of the appropriate category. The demand for labour in each of the group of production and management processes distinguished above is determined in a community break-down in each period by means of equations in the form of (10.7) and (10.8), while the input-output matrices of male and female labour (taking their reproduction, employment, and migration into account) are determined by the system of equations (10.9).

The variable $u_{i\mu}^{t}$ introduced into equations (10.9) above is explained as the number of labour of category μ in community i not employed in period t. This means that they are not being employed directly in the production and management processes being effected in each period, but may be employed in training personnel for operating new, improved machinery and equipment and controlling new technological processes, which is necessary in the conditions of the scientific and technical revolution. On the other hand, with a dynamic formulation of the problem of the forming of a complex, it is not excluded that it may be more profitable in certain conditions to maintain unutilised reserves than to lose them, if a growth in volume of production is planned in subsequent periods. In such cases these labour reserves are treated as the source for covering labour shortages [see variable $u_{i\mu}^{t-1}$ in constraint (10.9)].

The need to attract male and female labour to each community or a surplus of labour depends on the sex structure of the working people living in it, the increment in their numbers through natural growth, and the ratio of males and females among the workers arriving and departing. Some of the men attracted to the complex's facilities to cover a shortage of male labour arrive with families, and the female members of these families (if there is provision for their children in pre-school institutions) may take part in social production (additional reserves of female labour). Similarly, working women recruited to the complex may have males of employable age in their families. A reduction in the demand for male labour, and the departure of men caused by that, is associated with a reduction in the number of female workers, while a fall in the demand for female labour will entail the departure of a certain number of male workers. The constraints for observance of definite ratios of men and women in the number of workers arriving in a complex's communities and leaving them are represented in the model by the inequalities (10.10) to (10.13) inclusive.

The special features of the forming of a complex's population and manpower will not be sufficiently accurately taken into account unless the rate of change of its sex structure in each community is reflected in the model. The ensuring of definite proportions between male and female labour is provided for by constraints (10.14) and (10.15), which permit them to fluctuate within the limits $0_i^{t,\,\mathrm{min}}$ and $0_i^{i,\,\mathrm{max}}$.

The introduction of upper and lower limits is also possible in constraints (10.10) to (10.13) inclusive, by which the ratios of men and women among migrating workers are determined.

The data on the number of migrating workers in the period under review, the increase in the number of workers through natural increase, and the existence of a negative balance of migration in the preceding period serve as the basis in equations (10.16) for determining the volume of work on building the town social and amenity infrastructure and discovering its unutilised part for each community.

When the forming and development of a complex is treated dynamically, it is necessary to reckon with the fact that this process is not finished in the last of the periods reviewed, and that there will be a certain extension and improvement of production in subsequent years, growth of the population through natural increase, a raising of the standard of cultural amenities and services, and an improvement in the technical equipping of the facilities of this part of the infrastructure, and also major capital repairs to fixed assets. All that will call for the performance of a certain volume of building and assembly work. The numbers of building workers needed for these purposes are provided for by inequalities (10.17).

For sub-areals with limited land, water, and other natural resources a limit is set on the exploitation of each of them [see (10.18).

The separate variants of the development of town-forming objects may be linked with the exploitation of limited resources, for which a maximum volume of consumption is set by constraints (10.19). Such limitations (depending on the specific features of each resource) may take their consumption by the facilities of the town social amenity infrastructure into account, and may also be established in a break-down of the separate sub-areals. The limited scale of imports of labour into the complex to cover shortages in both town-forming objects and facilities of the town infrastructure, for instance, is taken into account by inequalities (10.21).

In providing for choice of the most efficient variants for developing objects in the course of optimising the forming of a complex, it is necessary to allow for the fact that the technical and economic characteristics proper to them hold only within a certain scale of application, the permissible limits of which are set by constraints (10.20).

Constraints (10.22) and (10.23) are written into the model as rigorous equalities. That means that we must count on the possibility, at the beginning of the period, of employing labour on the same objects, by the same variants, and in the same communities as in the preceding period. That approach is natural if there are no doubts about the expediency and efficiency of the actual distribution and employment of manpower. But it is quite permissible to treat the values P^{r0}_{ij}, $P^{0}_{i\mu}$, $\Pi^{0}_{i\mu}$, and $\Lambda^{0}_{i\mu}$ simply as upper limits of the application, distribution, and employment of manpower, and even, when feasible, to exclude them altogether or in part.

When the problem is solved by criterion (10.24), the following are taken into account: the operating costs of existing enterprises; the capital investment on building new town-forming objects and technically re-equipping previously built ones; the inputs on exploiting facilities of the town social and amenity infrastructure; the inputs on upkeeping idle fixed assets of the infrastructure associated with the maintenance of population of working age not employed on social production and with the migration of workers; and also inputs on transporting products.

Optimisation by criterion (10.25) provides information on the minimum irrational expenditure of time by the public and makes it possible to bring out the variant of the plan for forming the complex that most closely corresponds to the public's interest, in the context of the constraints considered.

The compromise plan (10.30) makes it possible, to some extent, to co-ordinate the interests of the economic authorities and those of the complex's population, and to determine at what price (in the conditions considered) a saving can be made in the time the public spends on commuting and journeys for cultural, household purposes and amenities.

The ways of building and operating the objects of the complex whose rate of application in the optimum plan depends on what criterion is used in the optimisation are distinguished from one another in the model (10.1) to (10.25) inclusive by the methods of developing the area, the forms of organising building and cultural and other social amenities and services, by work and rest regimes and other features that permit their joint realisation in one and the same object. This circumstance, and the non-integer character of most of the variables, allows us to conclude that the compromise plan obtained from the co-ordination will not be contradicted by the content of the means involved in it, and that its implementation will not give rise to special difficulties.

Deviations from the planned lines may arise during the practical implementation of the optimum plan for the forming of a complex, owing to the need to employ less efficient means of building and operating the objects, or to factors that were not taken into account when the problem was formulated. The future forming of the complex has to take these deviations into account, which calls for introducing appropriate amendments and adjustments into the optimum plan obtained earlier. Once the established position has been fixed by constraints of the type of (10.22) and (10.23), the optimum path for the complex's further development can be determined, allowing for all the deviations from the previously planned programme for its formation arising at this time, i.e. the model of the forming of a complex can also be employed as a tool of the process of controlling its building and functioning.

In this way the model of the forming of a complex makes it possible to take account of the main economic factors in the development of its economy, and of certain social factors, and can be employed to control the process of forming it.

Part III APPLIED STUDIES OF INDIVIDUAL PROBLEMS OF OPTIMISING THE FORMING OF A COMPLEX

11

THE SPECIFIC FEATURES OF TERRITORIAL INDUSTRIAL REGIONAL MESOMODELS

M. Yu. Cherevikina

In pre-planning studies for optimising the economy of territorial systems of mesolevel (economic region, intraregional territorial industrial combinations) the problem of determining the best variant for the spatial structure of each combination's economy is singled out.

Problems of this type arise during analysis of combinations of three ranks: namely, an economic region as a whole; a large territorial industrial complex within a region; and a separate industrial centre or agglomeration of centres. The different scales of the objects of research and the existence of specific features of each of them as a planning object necessitated the development of special mathematical economic models: namely, models of the spatial structure of the system of complexes in a region; models of a complex's spatial structure; and models of an industrial centre's spatial structure.

The main problem treated by means of these models is that of the location of industry. It is their industrial bias that differentiates these models from the others of the group that resolve problems of the dynamics of development, of the forming of elements of the infrastructure, and of migration and the population's standard of living, which can be joined with two other types of problem: namely, those of analysing the process of forming separate territorial industrial combinations, and those of determining the variant for shaping the separate elements of the infrastructure and the plan for building the combinations. The models for determining the best variant of the spatial structure of a combination's economy therefore belong to those of the territorial industrial type.

Problems of siting industry are also considered in sectoral (industrial) models, but models of the territorial industrial type differ from them in allowing for the aggregate effect of regional factors (infrastructure, local resources), with the result that the main territorial industrial combinations of a region are delineated together with the location of industry. Sectoral models have to pay greater attention to the specialisation of industry by output of some product by some technological method.

Problems of the location of industry in territorial systems are studied at macrolevel as well as at mesolevel. In spite of the fact, however, that the main economic elements of any territorial system are three groups of factors (industry, area, and manpower), the effect of each group on the development of systems of different level is not the same. At macrolevel, for instance, when the country as a whole functions as the territorial system, the greatest effect is exerted on it by the spatial functioning of industry, which is characterised by a difference in the coefficients of demand-output of products by regions, and by transport and the special (dual) role of population (which functions as both consumer and manpower). The territorial factors (local resources, infrastructure) affect a system's development indirectly, through the different spatial conditions of the development of industry and the provision of a certain standard of living for the population.

Production, as before, has a decisive effect on the development of territorial systems of mesolevel (regions, complexes). But the relationship between a system's groups of elements varies, above all, since its place in the territorial division of labour has already been determined, i.e. its deliveries of output of its specialisation industries are known, and the production links of these industries are not, as a rule, decisive in determining the spatial structure of a mesolevel system's economy. The effect of production consists already in the very necessity of its location in the given area. This formulation of the problem necessitates allowing for territorial elements in a different way and their representation as autonomous objects of study. At mesolevel population plays the role of local production reserves. The aim of such a territorial system is to reduce inputs on implementing the production programme set by the higher-standing system.

At microlevel (industrial centre, city) the relationship between the elements of territorial systems differs from that of the two foregoing levels. At this stage production is determined, in practice, and the infrastructure (production and social) and manpower (its settlement) begin to play the main role.

The territorial models describing systems of various level reflect the differences in relationship of the system's elements both in their structure and in the principles of their construction. Territorial industrial regional mesomodels differ essentially from models of the macro- and microlevels. The mathematical expression of the production constraints for optimising a multisector, inter-regional model (model of a higher level), for instance, is an intersectoral matrix, since a decisive role in the location of industries at that level is played by intersectoral connections. The mathematical expression of the production block of these models is an input-output type of limitation. The matrix characterising the production block, however, is not the intersectoral input-output table. In microlevel models the production block is transformed into a constraint on the capacity of production activities. The infrastructure and local resources, which are taken into account in models of a higher level through the coefficients of the intersectoral input-output matrix, are represented in mesomodels by two types of constraint: namely, (a) input-output matrices in which the formation (involvement) of the infrastructure (resource) is reflected; and (b) limitations from above on volume (rate of utilisation). The representation of the infrastructure (local resources) in microlevel models is similar to that in mesomodels but is more detailed.

A most important characteristic of a model is the mathematical form and economic sense of the objective function. The objective function in territorial industrial mesomodels is the minimum aggregate normalised inputs on the building and functioning of production units. It is characteristic that all the variables enter the function with non-zero coefficients, i.e. inputs are accounted for element by element. This form of objective function makes the links of the system more rigid and the properties of the models more definite, which corresponds to the premise adopted on the place of the mesolevel territorial systems in the economy's system. In the models of a higher level the objective function is the variable set of end products with a single coefficient. The problem is solved for the maximum of end product in the given production range. This form of function provides less rigid links between the system's elements and makes the model's properties less rigorous. The objective functions of micro- and mesomodels coincide, although others are also possible at microlevel (functions of maximum non-working time, of minimum travelling time, etc.).

Territorial industrial regional mesomodels are thus *spatial*, since the area is represented by several points (areals, industrial centres, location sites), and *optimising* models, whose objective functions are the minimum aggregate normalised inputs on carrying out the region's programme.

The special reflection of the effect of regional factors (infrastructure, and local resources) on the location of industry operates as the characteristic property of these mesomodels, the regional conditions of the location of industry being brought out in the special objects of investigation. The formalisation adopted in the models, with the singling-out of utilisation matrices and of limitations on the consumption of resources, makes it possible to allow for the effect of partial consumption of resources and to provide more detailed information about the location plan obtained.

The specific properties of territorial industrial models include a division of the objects of modelling by mode of comparison with the spatial system being studied. All the objects are divided into two groups, as follows:

(a) point objects—in exact conformity with the territorial unit of location being investigated, when the territorial unit of location is an attribute of the object; the objects include the production activities of the specialisation industries, facilities of the infrastructure (transport junctions, electricity sub-stations, facilities of the building industry like factories prefabricating concrete details, etc., treated as production activities), and local resources;

(2) linear objects—not conforming exactly with the territorial unit of location, i.e. compared with two or more objects of the spatial system; linear objects include the transport and power systems.

The output of the facilities of these groups may be mobile or non-mobile. In the first case (mobile output) there is an additional kind of variable that reflects the possible carriage of output between facilities of different units of location. These variables tie up the input-output matrices of the distribution of products with the constraints describing the shaping of the transport system. It should be noted, however, that formalisation is possible without identifying additional variables reflecting transport and economic links. Such a formalisation is expedient when the production activities being considered do not affect the change in the characteristics of the existing transport system.

Production and local resources (including labour) are represented in regional mesomodels by point objects with mobile output, in accordance with the classification adopted. The infrastructure may include objects of both types. All systems (transport and power) are classed as spatial objects, while all junctions or nodes, and some elements of the building industry, are treated as point objects. The elements of the building industry are treated either as production activities and represented in the model as enterprises or as local resources, i.e. point objects with non-mobile output, depending on the rank of the territorial-industrial combination.

The following notation is used when considering territorial industrial models.

INDICES AND SETS

j—index of production activities and enterprises located in the system (region, complex) ($j \in J$);

i, ν—indices of products and resources ($i, \nu \in I$; $I = \bigcup_{\alpha=1}^{4} I_\alpha, \quad \bigcap_{\alpha=1}^{4} I_\alpha = \varnothing$);

I_1—sub-set of the products of the specialisation industries;

I_2, I_3—sub-sets of products of complementary industries, I_2 being extractive industries, I_3 service industries;

I_4—sub-set of products brought in from outside the system and not produced within it;

k, p, k'—indices of the units of territory * (areal, industrial centre); $(k, p \in K)$;

$\varphi\psi$—indices of elements of the infrastructure $(\psi, \varphi \in \in \Phi; \varphi \in \Phi_1 \cup \Phi_2)$;

Φ_1— elements of the infrastructure relating to transport;

Φ_2—elements of the infrastructure relating to power;

n—objective variant of the use of land and water resources (used for industrial and civil building, farming, etc.) $(n \in N)$;

μ, μ'—categories of labour $(\mu, \mu' \in M)$;

r—variants of functioning, utilisation, input

$$(r \in R;\ R = \bigcup_{\alpha=1}^{12} R_\alpha,\ \bigcap_{\alpha=1}^{12} R_\alpha = \varnothing);$$

R_1—sub-set of variants of the functioning of production activities (enterprises);

R_2—sub-set of variants of the functioning of the building industry;

R_3—sub-set of variants of the working of deposits;

R_4—sub-set of variants of the functioning of local power systems;

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* The set of areals (industrial centres) is sub-divided as follows, depending on the way of reflecting transport-production links in the models, into:
(a) a 'network' representation of transport links $K = K_1 \cup K_2$, in which K_1 is the sub-set of the boundary areals of the region through which the system's imports and exports are made, and K_2 is the sub-set of interior areals;
(b) fixed transport economic links reflected in the variants of a production activity's (enterprise's) development $K = K_j \cup M_i$, in which K_j is the sub-set of areals (industrial centres) in which production activity (enterprise) j is located, and M_i is the sub-set of areals (industrial centres) in which deposit i of raw materials is located.

R_5—sub-set of variants of the development (of section pk) of power transmission lines;

R_6—sub-set of variants of the reconstruction of a single-track section (pk) of the transport system;

R_7—sub-set of variants of the building (reconstruction) of a single-track section (pk) of the transport system;

R_8—sub-set of variants of the functioning of transport junctions;

R_9—sub-set of variants of the functioning of heat sources;

R_{10}—sub-set of variants of the grading of land resources;

R_{11}—sub-set of variants of the grading of water resources;

R_{12}—sub-set of variants of the development of a town.

UNKNOWNS

x_{jk}^r—rate of functioning of production activity (enterprise) j located in areal (industrial centre) k, by method r ($x_{jk}^r \geqslant 0$, $j \in J$; $k \in K$; $r \in R_1 \cup R_2 \cup R_3$);

x_{ipk}—volume of deliveries of product (resource) i from areal (industrial centre) p to areal (industrial centre) k ($i \in I$; $k \in K$; $x_{ipk} \geqslant 0$);

x_{ik}—import of product (resource) i from outside the territorial industrial combination to areal (industrial centre) k ($i \in I$; $k \in K$; $x_{ik} \geqslant 0$);

$z_{kp}^{r\varphi}$—rate of functioning of element φ of the spatial infrastructure on section kp of a system by variant r ($\varphi \in \Phi$, k, $p \in k$, $r \in R_5 \cup R_6 \cup R_{11}$, $z_{kp}^{r\varphi} \geqslant 0$);

$z_k^{r\varphi}$—rate of functioning of element φ of the point infrastructure in areal (industrial centre) k by variant r ($\varphi \in \Phi$, $k \in K$, $r \in R_4 \cup R_8 \cup R_9$, $z_k^{r\varphi} \geqslant 0$);

u_{nk}^r—amount of land (water) of grading variant r utilised for purpose n in areal (industrial centre) k ($k \in K$; $n \in N$; $r \in R_{10} \cup R_{11}$, $u^r_{nk} \geqslant 0$);

l^{μ}_{k}—amount of local category μ labour employed in the production activities of areal (industrial centre) k ($k \in K$, $\mu \in \mathrm{M}$, $l^{\mu}_{k} \geqslant 0$);

$\hat{l}^{\mu}_{k}$—amount of category μ labour drawn to areal (industrial centre) k from outside the region ($k \in K$, $\mu \in \mathrm{M}$, $\hat{l}^{\mu}_{k} \geqslant 0$);

l^{r}_{k}—size (population) of a town of category r ($k \in K$; $r \in R_{12}$; $l^{r}_{k} \geqslant 0$);

$\tilde{l}^{\mu}_{k}$—numbers of category μ labour living in industrial centre k and not employed in the industries considered ($k \in K$; $\mu \in \mathrm{M}$; $\tilde{l}^{\mu}_{k} \geqslant 0$).

MATRIX COEFFICIENTS AND LIMITATIONS

A^{r}_{ij}—volume of output of product i by enterprise (production activity) j, functioning by variant r ($j \in J$; $i \in I$; $r \in R_1 \cup R_2 \cup R_3$; $A^{r}_{ij} = 0$, if $i \in I_4$; $A^{r}_{ij} > 0$ in other cases);

$\beta^{r}_{i\nu}$—consumption of product (resource) i in producing a unit of product ν by technological variant r ($i, \nu \in I$; $r \in R_1 \cup R_2 \cup R_3$; $\beta^{r}_{i\nu} = 0$, if $i \in I_1 \vee \nu \in I_4 \vee i \equiv \nu$; $\beta^{r}_{i\nu} > 0$ in other cases);

d_{ik} —maximum possible volume of deliveries of product (resource) i from outside the region (complex) to areal (centre) k ($k \in K$; $i \in I$; $d_{ik} > 0$, when $i \in I_4 \wedge k \in K_1$);

B_{ik}—target for shipment of product (resource) i out of the region (complex) through areal (centre) k ($k \in K$; $i \in I$; $B_{ik} > 0$, when $i \in I_1 \cup I_2 \wedge k \in K_1$);

$\Pi^{r\varphi}_{kp}$—carrying capacity of section (kp) of element φ of the infrastructure under variant r ($k, p \in K$; $\varphi \in \Phi$; $r \in R_5 \cup R_6 \cup R_7$; $\Pi^{r\varphi}_{kp} > 0$);

Π^{φ}_{i}—input of the product of element φ of the infrastructure per unit of product i ($i \in I$; $\varphi \in \Phi$; $\Pi^{\varphi}_{i} > 0$, when $\varphi \in \Phi_1$; $\Pi^{\varphi}_{i} = 0$ in other cases);

$Э_j^{r\varphi}$—input of the ‘product’ of element φ of the infrastructure in production activity j functioning by variant r ($\varphi \in \Phi$; $j \in J$; $r \in R_1 \cup R_2 \cup R_3$; $Э_j^{r\varphi} > 0$, with $\varphi \in \Phi_2$; $Э_j^{r\varphi} = 0$ in other cases);

$Э_{kr}^{\varphi\psi}$—input of the ‘product’ of element φ of the infrastructure in element ψ functioning by variant r in areal (centre) k ($k \in K$; $\varphi, \psi \in \Phi$; $Э_{kr}^{\varphi\psi} = 0$ when $\varphi = \psi$; $Э_{kr}^{\varphi\psi} > 0$ in other cases);

π^φ—the population's consumption of the product of element φ of the infrastructure ($\varphi \in \Phi$; $\pi^\varphi > 0$);

$P_k^{r\varphi}$—capacity of variant r of the functioning of element φ of the point infrastructure located in areal (centre) k ($k \in K$; $\varphi \in \Phi$; $r \in R_4 \cup R_8 \cup R_9$; $P_k^{r\varphi} > 0$);

S_{jn}^r—amount of land (water) needed for the functioning of production activity (enterprise) j by variant r ($j \in J$; $n \in N$; $r \in R_1 \cup R_2 \cup R_3$; $S_{jn}^r > 0$);

$S_{kn}^{r\varphi}$—amount of land (water) needed for the building and functioning by variant r of element φ of the point infrastructure in areal (centre) k ($k \in K$; $n \in N$; $\varphi \in \Phi$; $r \in R_4 \cup R_7 \cup R_8$; $S_k^{r\varphi} > 0$);

s_n—amount of land (water) needed to provide services for the population ($n \in N$; $s_n > 0$);

S_k^r--amount of land of r grade in areal (centre) k ($k \in K$; $r \in R_{10} \cup R_{11}$; $S_k^r \geqslant 0$);

$T_{j\mu}^r$—amount of labour of category μ needed for the functioning of production activity (enterprise) j by variant r ($j \in J$; $r \in R_1 \cup R_2 \cup R_3$; $\mu \in \mathrm{M}$, $T_{j\mu}^r \geqslant$ $\geqslant 0$);

$T_{k\mu}^{r\varphi}$—amount of labour of category μ needed for the functioning of variant r of element φ of the infrastructure in areal (centre) k ($k \in K$; $\varphi \in \Phi$; $\mu \in \mathrm{M}$; $r \in R_4 \cup R_8$; $T_{k\mu}^{r\varphi} \geqslant 0$);

T_k^μ—reserve of local labour of category μ in areal (centre) k ($k \in K$; $\mu \in \mathrm{M}$; $T_k^\mu \geqslant 0$);

$\alpha_{\mu\mu'}$—coefficient of the ratio between μ and μ' reserves of labour ($\mu, \mu' \in M$; $\alpha_{\mu\mu'} > 0$);

γ—coefficient of the size of families ($\gamma > 0$).

FUNCTION DESIGNATORS

c^r_{jk}—normalised inputs* on the building and functioning of production activity (enterprise) j by variant r in areal (centre)** k ($k \in K$; $j \in J$; $r \in R_1 \cup R_2 \cup R_3$; $c^r_{jk} > 0$);

c_{ikp}—input linked with transporting a unit of product i over section (kp) of the transport system ($k, p \in K$; $i \in I$; $c_{ikp} > 0$);

$\hat{c}_{ik}$—input associated with making product i outside the territorial system under consideration and delivering it to areal (centre) k ($i \in I$; $k \in K$; $\hat{c}_{ik} \geqslant 0$; $\hat{c}_{ik} > 0$, when $i \in I_4 \wedge k \in K_1$);

$c^{r\varphi}_{kp}$—input entailed in the building and functioning of element φ of the spatial infrastructure on section (kp) under variant r ($k \in K$; $\varphi \in \Phi$; $r \in R_5 \cup R_6 \cup R_7$; $c^{r\varphi}_{kp} > 0$);

$c^{r\varphi}_k$—input connected with the building and functioning of element φ of the point infrastructure in areal (centre) k under variant r ($k \in K$; $\varphi \in \Phi$; $r \in R_5 \cup R_6 \cup R_7$; $c^{r\varphi}_k > 0$);

c^r_{nk}—input connected with the preparation of land (water resources) of grading variant r for purpose n in areal (centre) k ($k \in K$; $n \in N$; $r \in R_{10} \cup R_{11}$; $c^r_{nk} > 0$);

c^μ_k—input connected with drawing local category μ labour into production and providing it with services in areal (centre) k ($\mu \in M$; $k \in K$; $c^\mu_k > 0$);

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* Henceforward we shall refer to normalised inputs simply as input.

** With a variant of the means of reflecting transport and economic links, c^r_{jk} includes inputs on transporting the output.

$\hat{c}_k^\mu$—input connected with attracting category μ labour from outside the region (complex) and settling it in areal (centre) k ($k \in K$; $\mu \in \mathrm{M}$; $\hat{c}_k^\mu > 0$);

c_k^r—input connected with building a town of category r in industrial centre k ($k \in K$; $r \in R_{12}$; $c_k^r > 0$);

$\tilde{c}_k^\mu$—input connected with attracting μ-category labour to areal (centre) k so as to meet the structural ratio ($k \in K$; $\mu \in \tilde{\mathrm{M}}$; $\tilde{c}_k^\mu > 0$).

In accordance with the different character of the objects considered, territorial industrial models can be broken down into logic blocks with a different structure: namely, production block, blocks of the infrastructure, local resource and labour (see Diagram 11.1).

For the production block of a territorial industrial model it is characteristic to distinguish between the production activities of the specialisation and complementary industries, as objects of investigation, and the facilities of the infrastructure equated with production activities.

The block consists of three types of equations, as follows:

(i) input-output matrices of the production and distribution of products by areals (centres, sites), reflecting the possibility of producing output of a certain type at several facilities, demand within the areal, the transport and technological links between the various territorial units of the system being studied, and connections regarding imports and exports (imports being an exogenous parameter);

(ii) limitations on the choice of variant of the functioning of production activities*;

(iii) constraints on the variable volume of imports of scarce resources and products from outside the territorial system being studied.

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* The economic sense of this constraint lies in its requiring choice of one variant only of the functioning of a production activity, but without a whole-number formulation of the model this requirement is not met.

The production block of a standard model can be represented as follows:

$$\sum_{j\in J}\sum_{r\in R} A^r_{ij}x^r_{jk} - \sum_{j\in J}\sum_{r\in R}\sum_{v\in I} \beta^r_{iv}A^r_{vj}x^r_{jk} + \\ + \sum_{p\in K} x_{ipk} - \sum_{p\in K} x_{ikp} + \hat{x}_{ik} \geqslant B_{ik} \\ (i\in I,\ k\in K); \qquad (11.1)$$

$$-\sum_{k\in K}\sum_{r\in R} x^r_{jk} \geqslant -1\ (j\in J); \qquad (11.2)$$

$$-\hat{x}_{ik} \geqslant -d_{ik}\,(i\in I,\ k\in K). \qquad (11.3)$$

The production blocks of concrete territorial industrial mesomodels are modified according to the specific features of the rank of combination. In a model of the spatial structure of a region's system of complexes, it is broken down into a directly production block and a block of the region's raw material base. In the former the objects of investigation are the major aggregated production activities of the specialisation and complementary industries, each of which is represented by methods that describe its possible functioning. The vector corresponding to a method contains indices of output (which is also aggregated); and the input of mineral and local resources. The indices of the functioning of production activities are treated as converted into standard sectoral capacities; consequently, they do not depend on the index of the areal. The targets for shipping out the product of the specialisation industries operate as the vector of the constraints in the block under consideration. Such targets do not exist for the complementary industries. The production block takes the following form:

$$\sum_{r\in R_1}\sum_{j\in J} A^r_{ij}x^r_{ik} - \sum_{v\in I}\sum_{j\in J}\sum_{r\in R_1} \beta^r_{iv}A^r_{vj}x^r_{jk} + \\ + \sum_{p\in K} x_{ipk} - \sum_{p\in K} x_{ikp} + \hat{x}_{ik} \geqslant \\ \geqslant B_{ik}\ (i\in I,\ k\in K)\ \text{(shadow prices } u_{ik}\text{)}.$$

Since the model's criterion is the minimum summed normalised input, i.e. overfulfilment of the programme does not profit the region, the inequalities of the production block are converted into equalities in the optimum plan, the shadow prices of the output of the specialisation industries being positive (see Chapter 12).

The constraints describing the functioning of the region's raw material base are similar to those describing the functioning of the production activities of the specialisation and complementary industries. But in this case there is a limitation on the variable possible volume of extraction of raw material of type i:

$$x_{im}+\hat{x}_{im}-\sum_{j\in J}\sum_{r\in R_1}\sum_{\nu\in I}\beta^r_{i\nu}A^r_{\nu j}x^r_{jk}-\sum_{p\in K}x_{imp}+\sum_{p\in K}x_{ipm}\geqslant 0$$

$$(i\in J,\ m\in K);$$

$$x_{im}\leqslant A_{im};\ \hat{x}_{im}\leqslant d_{im}.$$

Another formalisation of this constraint is possible: namely, without singling out the variable x_{im} and the additional limitation on it. Analysis has shown that the notations used in the models of the spatial structure of a region's system of complexes also make it possible to obtain additional information on the effect of the raw material base on the location of production activities with underutilisation of the possible volume of extraction.

In the model of a complex's spatial structure, in contrast to that of a region's system of complexes, the objects of investigation of the production block are not large aggregated production activities but enterprises. This necessitates taking account of the several technological variants that are found when sectoral models of the region are solved. The degree of disaggregation, and the treatment of actual economic objects at complex level, require allowing for the interchangeability of products, and for output of one and the same product at several enterprises. The problem of taking account of the production of an assortment of products at one enterprise becomes extremely important. These aspects find reflection in the variants of the functioning of enterprises.

At regional level the variants of the production methods in the model have quite a different economic sense. During investigation of large aggregated production activities within such taxonomic units as economic regions, the foreground is taken up not by the technological variant in the sectoral meaning of the term, i.e. not by the actual technology of the production of a certain item with all its technical and economic indices (its treatment does not make sense in relation to aggregated production activities and products), but the fundamentally different projects for technologies differing in capital and labour intensity, and possibly in raw materials and power.

It is possible, however, that when a model of a region is being constructed in which the strategy of its development is decided, the need to consider problems of the capital and labour intensity of the functioning of production activities will lose its point when the spatial specialisation of the system of complexes is being considered. In fact, in the conditions of the given production structure of a region's development, these factors affect only the size of the total input on carrying out an already known programme. But when labour is considered as settled in certain areals, i.e. when intraregional migration is not considered in the model, the different labour intensity of the functioning of production activities can affect the structure of location. Thus the sense of some of the parameters of one type of model may be altered, depending on the rank of the territorial system.

With all these peculiarities, the reflection of the specialisation industries in both models represents limitations constructed in the form of an input-output matrix for each areal (sub-areal) and enterprise (product).

The complementary industries are represented in both models in a similar way, while retaining all the differences in notations of the specialisation industries.

The production block of an individual complex has the following form:

$$\sum_{j,\,k,\,r} a_{ij}^{r} x_{jk}^{r} - \sum_{k,\,j,\,\nu,\,r} \beta_{i\nu}^{r} a_{\nu j}^{r} x_{jk}^{r} \geqslant 0 \; (i \in I); \qquad (11.4)^{*}$$

$$-\sum_{k,\,r} x_{ik}^{r} \geqslant -1 \; (j \in J); \qquad (11.5)$$

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*The content of indices a_{ij}^{r} is similar to A_{ij}^{r}, but a_{ij}^{r} corresponds to unit volume rather than to the capacity of a standard enterprise.

$$\sum_{r \in R_3} x_{im}^r - \sum_{\nu,\, k,\, r} \alpha_{i\nu}^r b_{\nu j}^r x_{ik}^r > 0 \ \ (i \in I,\ m \in M); \qquad (11.6)$$

$$-\sum_r x_{im}^r \geqslant -A_{im}^r \ \ (i \in I,\ m \in M). \qquad (11.7)$$

Even with a cursory comparison of the production blocks of models of the spatial structure of a region and of a complex, we can notice a difference in the means of representing facilities' transport and economic links. This is a second important aspect of the formalisation of the production block of territorial industrial mesomodels. In the model of a complex's spatial structure industrial centres do not have a transport division into external and interior; instead, enterprises are distinguished by variants of their transport and economic links with suppliers and customers. That way of representing these links reduces the number of equations in the model, but it is only legitimate when the production activities considered do not affect the shaping of the transport system's configuration or its carrying capacity.

The fixing of transport links in variants of location (as in the model of a complex's spatial structure) or their free choice (as in the model of the spatial structure of a region's system of complexes) thus does not depend on the rank of the territorial system or combination being studied but on the effect of the located industries on the shaping of the transport system, the process of which is reflected in the infrastructure block.

In the infrastructure block of territorial industrial mesomodels elements of the linear infrastructure (transport and power systems) and of the point infrastructure (spatial centres, junctions, and power centres) are treated as objects. Two sub-blocks are accordingly distinguished. The first consists of equations of three types, as follows:

(1) an input-output matrix of the forming of the load on a section of the transport system and choice of its rate of functioning; the rate is selected in relation to the needs of production activities, the population, the point facilities of the infrastructure and transport, and to technological connections; the limitations of this type are thus linked with the production block and with the manpower block and the point infrastructure block;

(2) limitations on the linear combination of variable rates of functioning of the system's sections;

(3) choice of the synonymous variant of the opposite directions of the system's sections:

$$\sum_{r\in R} \Pi_{kp}^{r\varphi} z_{kp}^{r\varphi} - \sum_{i\in I} \Pi_{i}^{\varphi} x_{ikp} - \sum_{j\in J} \sum_{r\in R} Э_{j}^{r\varphi} x_{jk}^{r} -$$

$$- \sum_{\psi\in\Phi} \sum_{r\in R} Э_{kr}^{\varphi\psi} z_{k}^{r\varphi} - \pi^{\varphi} \sum_{\mu\in M} (l_{k}^{\mu} + \hat{l}_{k}^{\mu}) \geqslant 0 \quad (\varphi\in\Phi,\ p,\ k\in K); \tag{11.8}$$

$$- \sum_{r\in R} z_{kp}^{r\varphi} \geqslant -1 \quad (\varphi\in\Phi,\ k,\ p\in K); \tag{11.9}$$

$$\sum_{r\in R_7} z_{kp}^{r\varphi} - \sum_{r\in R_7} z_{pk}^{r\varphi} \geqslant 0 \quad (\varphi\in\Phi,\ p\in K). \tag{11.10}$$

The second sub-block consists of equations of two types: namely,

(1) input-output matrices of the choice of rate of functioning of a point object of the infrastructure (transport junction or local power system), the constraint being linked with the first sub-block of the infrastructure;

(2) limitations on the choice of variant of the rate of functioning of the transport or power system:

$$\sum_{r\in R} P_{k'}^{r\varphi} z_{k'}^{r\varphi} - \sum_{r\in R} \sum_{p\in K} \sum_{k\in\{k-k'\}} \Pi_{kp}^{r\varphi} z_{kp}^{r\varphi} \geqslant 0 \, (\varphi\in\Phi,\ k\in K); \tag{11.11}$$

$$- \sum_{r\in R} z_{k}^{r\varphi} \geqslant -1 \, (k\in K,\ \varphi\in\Phi). \tag{11.12}$$

The building industry, though an element of the infrastructure, is not treated in the infrastructure block but in the production block, and is included in the category of auxiliary industries; in concrete models the infrastructure block is broken down into three sub-blocks: namely, transport, power, and the building industry.

In order to reflect transport the existing carrying capacity of a section of the system is treated typically as a limitation, since variants of its development are being considered. In addition, the development variants make it possible to some extent to allow for the non-linear character of the inputs on creating another carrying capacity. The load on the section is formed in the production block, while the variant is selected in the transport development block in relation to the location scheme obtained. There are thus two groups of variables characterising transport: namely, the volume of freight and the intensity (carrying capacity) of the variant for the development of a section of the transport system.

The first group of variables in the model of the spatial structure of a region's system of complexes includes special variables z^r_{kp} in connection with the free reflection of transport and economic links adopted. In the model of a complex's spatial structure the volumes of freight traffic are determined in proportion to the output of products through coefficients reflecting the economic links of production activities $\Pi^r_{jkp}x^r_{jk}$. The second group of variables consists of z^r_{kp} in both models.

The constraints put into the transport block include an input-output matrix of the satisfaction of demand, choice of a variant of the section's functioning, and choice of a synonymous variant for both directions of the section:

(i) in the model of the spatial structure of a region's system of complexes

$$\sum_{r\in R}\Pi^r_{kp}z^r_{kp}-\sum_{i\in I}\Pi_i x_{ikp}\geqslant 0 \quad (k,\ p\in K)\ (\text{shadow prices } \xi_{kp});$$

$$-\sum_{r\in R}z^r_{pk}\geqslant -1\ (p,\ k\in K) \quad (\text{shadow prices } \gamma_{pk});$$

$$z^r_{pk}-z^r_{kp}\geqslant 0 \quad (p,\ k\in K,\ r\in R_6) \quad (\text{shadow prices } \zeta^r_{pk});$$

(ii) in the model of a complex's spatial structure

$$\sum_{r\in R}\Pi^r_{kp}z^r_{kp}-\sum_{j,\ r}\Pi^r_{jkp}x^r_{jk}\geqslant 0 \quad (k,\ p\in K) \quad (\text{shadow prices } \xi_{kp});$$

$$-z^r_{kp}\geqslant -\Pi^r_{kp}\,(k,\ p\in K) \quad (\text{shadow prices } \gamma^r_{pk}).$$

In contrast to sectoral models no limitation on the volume of traffic is considered in territorial industrial models, but the development of the transport system is treated in dependence on the demand formed in the production block. This method makes it possible, in addition to determining the variant for development of a section of the system, to obtain data also (in the event of underutilisation of the section's carrying capacity) on the role of the given section in shaping the optimum plan, since the shadow price of exploiting it always remains positive in the conditions of its operation. In addition, the shadow price of the exploitation of a section is a component of the shadow price of output in an areal. Transport thus affects location even when the former is underutilised.

The shadow prices included in the transport block of these mesomodels reflect the total inputs of the system concerned on the creation and functioning of transport. Their structure is quite simple and reasonable. The corresponding blocks of the dual problems take the following form:

(i) in the model of the spatial structure of a region's system of complexes

$$\sum_{i} A^r_{ij} u_{ik} - \sum_{r \in R_1} \sum_{\nu \in I} \beta^r_{\nu i} A^r_{\nu j} u_{\nu k} \leqslant c_{jk} \quad (j \in J,\ k \in K,\ r \in R)$$

(shadow prices x^r_{jk});

$$\Pi^r_{pk} \xi_{pk} - \gamma^r_{pk} + \zeta^r_{pk} - \zeta^r_{kp} \leqslant c^r_{kp} \quad (p,\ k \in K;\ r \in R_6)$$

(shadow prices z^r_{pk});

$$u_{ik} - u_{ip} - \xi_{pk} \leqslant c_{ipk} \quad (i \in I,\ p,\ k \in K); \text{ (shadow prices } x_{ipk});$$

(ii) in the model of a complex's spatial structure

$$\sum_{i \in I} a^r_{ij} u_i - \sum_{\nu,\ r \in R_1} \beta^r_{\nu i} a^r_{\nu j} u_\nu - \Pi^r_{jkp} \xi_{kp} \leqslant c_j \quad (j \in J;\ r \in R,\ k \in K)$$

(shadow prices x^r_{jk});

$$\xi_{kp} - \gamma^r_{kp} \leqslant c^r_{kp} \quad (k,\ p \in K;\ r \in R) \text{ (shadow prices } z^r_{kp}).$$

Given utilisation of section (kp) in the optimum plan, the shadow price of the variant of carrying capacity included in the plan equals the sum of the inputs on creating the corresponding carrying capacity and the shadow prices of exploitation of the section:

$$\gamma^r_{kp} = c^r_{kp} + \xi_{kp}.$$

This relationship is satisfied in both models, as is confirmed by practical calculations.

The shadow price of exploiting a section of the transport system in territorial industrial mesomodels reflects the relationship between inputs on producing output at the points of production and consumption, taking account of the demand for it at both points, and the direct inputs on transportation. In the model of a region this relationship takes the following form:

$$\xi_{pk} = u_{ik} - u_{ip} - c_{ipk},$$

in the model of a complex it has the following form:

$$\Pi^r_{jpk}\xi_{pk} = \sum_{i\in I} a^r_{ij}u_i - \sum_{\nu,\, r} \beta^r_{\nu i}a^r_{\nu j}u_\nu - c_j.$$

The other two elements of the spatial structure, i.e. the building industry and the power system, are reflected in the models in the same way as transport.

When the functioning of the building industry is being determined, as we have already noted, the point elements are singled out (their reflection is equivalent to that of enterprises, and are consequently included in the production block), and spatial elements (i.e. building sites), which are represented by the flows of services performed in various areals (centres) by a certain building industry or firm. As in the transport sub-block, the rate (volume) of building work for which it is necessary to provide a building industry, and the flows supplying each of the location points, are determined in the building industry sub-block. Problems of the dynamics of the building of facilities are not yet considered in the mesomodels thus the purpose of singling out the building industry and its location as an autonomous object is to take account of the costs of the location of production activities element by element, while the formalisation of the building industry is similar to that of other spatial elements of the infrastructure:
(i) in the model of the spatial structure of a region's system of complexes

Primal solution	Dual solution
$\sum_{r,\, q} y^r_{qk}\pi^r_q - \sum_{j,\, r} \pi^r_j x^r_{jk} - \gamma\pi \times$	$\pi^r_q\alpha_k - \beta_q \leqslant c^r_{qk}$
$\times \sum_{\mu} (l^\mu_k + \hat{l}^\mu_k) \geqslant 0 \; (k \in K)$	$(q \in Q,\; r \in R,\; k \in K)$

(shadow prices α_k);

$$-\sum_{k,\,r} y^r_{qk} \geqslant -1\,(q \in Q)$$

(shadow prices β_q);

$$\sum_{r,\,j} A^r_{ij}x^r_{jk} - \sum_{\nu,\,r,\,j} \beta^r_{i\nu}A^r_{\nu j}x^r_{jk} -$$
$$-\sum_p x_{ikp} + \sum_p x_{ipk} + \hat{x}_{ik} \geqslant B_{ik}$$
$$(i \in I;\ k \in K)$$

(shadow prices u_{ik});

(shadow prices y^r_{qk});

$$\sum_{i \in I} A^r_{ij}u_{ik} - \pi^r_{jk}\alpha_k \leqslant c^r_{jk}$$
$$(i \in J,\ k \in K;\ r \in R)$$

(shadow prices x^r_{jk});

(ii) in the model of the spatial structure of a complex

Primal solution

$$\sum_{r,\,q} y^r_{qk} - \sum_{j,\,r} \pi^r_j x^r_{jk} - \gamma\pi \times$$
$$\times \sum_\mu (l^\mu_k \times \hat{l}^\mu_k) \geqslant 0\,(k \in K)$$

(shadow prices α_k);

$$-y^r_{qk} \geqslant -\pi^r_q\,(q \in Q;\ r \in R)$$

(shadow prices β^r_q);

$$\sum_{j,\,k,\,r} a^r_{ij}x^r_{jk} - \sum_{k,\,j,\,r} \beta^r_{i\nu}a^r_{\nu j}x^r_{jk} \geqslant 0\,(i \in I)$$

(shadow prices u_i);

Dual solution

$$\alpha_k - \beta^r_q \leqslant c^r_{qk}$$
$$(q \in Q,\ k \in K;\ r \in R)$$

(shadow prices y^r_{qk});

$$\sum_i a^r_{ij}u_i - \pi^r_{jk}\alpha_k \leqslant c^r_{jk}$$
$$(j \in J,\ k \in K)$$

(shadow prices x^r_{jk})

The building industry gets two shadow prices in the optimum plan in these mesomodels: namely, α_k, the shadow price of its exploitation in areal (industrial centre) k; and β_q, the shadow price of the industrial capacity. In both models the shadow price of the building industry entering the optimum plan is built up from the direct costs of its creation and the shadow price of its exploitation. The difference is that, in the first case, the price is a unit value and in the second case it applies to the whole volume. This difference has no association with the rank of the territorial systems being studied. The same remarks about the effect of the building industry on the location of production activities apply to its sub-block as apply to the transport sub-block.

The reflection of the power system pursues the aim of accounting for inputs element by element. The structure of its sub-block, shadow prices, and effect on the location of production activities have the same character as those of transport and the building industry.

Manpower has a vital effect on the location of industry. When the spatial organisation of a mesosystem is being determined labour functions only as a production factor, but that does not mean that the model does not, in general, take the population's interests into account. Inputs on providing services for the labour force, and for its families, and on the population's needs for local resources and the infrastructure are reflected in the models. This treatment may seem inadequate, since many problems of reproduction of the labour force are not taken into account: namely, the birth rate, migration, training, determination of the structure and size of the population's needs and the satisfying of the same. These problems are differentiated from those of the area's spatial organisation, although they are closely interconnected. The treatment of labour in our mesomodels as one of the factors in the development of production is therefore quite justified, especially when attention is paid to the fact that the production need itself necessitates taking several aspects of the reproduction of manpower into account. The location of industry, for instance, may give rise to a migration of population, or to a change in its occupational and sex structure, while the attraction of manpower to districts of new development entails a need to provide it with housing and services.

The manpower block consists of equations of the following three types: (1) balances of manpower of certan categories of labour at each site of industry, balances that are linked with both the production and the infrastructure blocks; the demand for labour may be met from the local population and by people attracted from outside the system (combination) concerned; (2) limitations on the variable use of local manpower, taking its structure into account; (3) input-output matrices of the satisfying of the structural ratio between the different categories of labour attracted:

$$l_k^{\mu} + \hat{l}_k^{\mu} - \sum_{r\in R}\sum_{j\in J} T_{j\mu}^{r} x_{jk}^{r} - \sum_{r\in R}\sum_{\varphi\in\Phi} T_{k\mu}^{r\varphi} z_k^{r\varphi} \geqslant 0 \ (k\in K;\ \mu\in M); \tag{11.13}$$

$$-l_k^{\mu} \geqslant -T_k^{\mu} \quad (\mu\in M;\ k\in K); \tag{11.14}$$

$$\hat{l}_k^{\mu} - \alpha_{\mu\mu'} \hat{l}_k^{\mu'} + \tilde{l}_k^{\mu} - l_k^{\mu'} \geqslant 0 \quad (k\in K;\ \mu\in M); \tag{11.15}$$

The constraints describing manpower in concrete models do not differ in form of notations (like all the other blocks of both models), but do differ in the number of factors considered at the level of a region or complex, and consequently in the number of variables and constraints.

In the model of the spatial structure of a region's system of complexes, for instance, the manpower of an areal functions as an object of investigation without classification by sex, age, or skills (which is not necessary at the given stage of planning). But the problem of accounting for migration arises already at this stage. Territorial industrial mesomodels are some of the few types of model that allow for the conditions of attracting labour from outside the region (variable $\hat{l}_k^{\mu}$). The possible exodus of labour is not accounted for so well—through an increase in the coefficient of the needs of industry, i.e. virtually through estimated shadow prices.

In the model of the spatial structure of a region's system of complexes the manpower block includes input-output matrices of the utilisation of labour, and constraints on the use of reserves of local labour:

$$l_k + \hat{l}_k - \sum_{j,r} T_j^r x_{jk}^r - \sum_{q,r} T_{qk}^r y_{qk}^r \geqslant 0 \ \ (k\in K) \ \ (\text{shadow prices } \omega_k);$$

$$-l_k \geqslant -T_k \ \ (k\in K) \ \ (\text{shadow prices } \theta_k).$$

The isolation of a variable l_k in the model, i.e. the utilisation of local manpower, is justified in this case since it helps provide additional information about the effect of regional conditions on the location of industry, and also allows for their effect with partial utilisation of reserves.

The manpower of areal k, employed in the production activities concerned, like production, functions as a consumer of local resources and of the infrastructure. Manpower, for example, is involved in the shaping of the demand for building work:

$$\sum_{r,q} y^r_{qk}\pi_q - \sum_{j,r} \pi^r_{jk}x^r_{jk} - \gamma\pi\,(l_k + \hat{l}_k) \geqslant 0 \quad (k \in K)$$

(shadow prices α_k).

The shadow price of the drawing of local labour into production in areal k (ω_k) is formed taking account of the shadow prices of the exploitation of local resources and of the infrastructure:

$$\omega_k = c_k + \theta_k + \gamma\pi\alpha_k,$$

that is to say, when local labour ($l_k > 0$) is employed in an areal, the shadow price of drawing it into production is equal to the sum of the shadow prices of the exploitation of local resources and of the infrastructure in the areal plus the shadow prices of the manpower reserve and the inputs on providing it with services. In this way the close interdependence of territorial resources is reflected in this type of mesomodel.

The shadow price of the reserve of local labour (θ_k) does not exceed (and in the optimum plan is equal to) the difference between the inputs on providing services for the labour attracted from outside the region and for local manpower.

$$\theta_k \leqslant \grave{c}_k - c_k.$$

Labour can thus be brought into an areal if the inputs on attracting and settling it do not exceed the sum of the shadow of drawing local labour into production and the inputs on providing it with services.

The same approach is used in the model of a complex's structure as was in the foregoing model to formalise the process of providing industry with labour. But at the level of planning concerned the requirements for modelling manpower are rather different. When enterprises are involved, and not standard aggregated production activities, manpower is no longer uniform. In the model of a complex the composition of the labour force is given by sex. It is probably necessary to give the occupational structure (by skills) (albeit aggregated). As a result, this model, in contrast to that of the spatial structure of a region, has additional variables l_k and $\hat{l}_k$, which signify the type of labour, while the constraints describing the functioning of labour are supplemented by a limitation reflecting the ratio between male and female labour:

$$\hat{l}_k^{\mu} - \alpha_{\mu\mu'}\hat{l}_k^{\mu'} + \tilde{l}_k^{\mu} - \tilde{l}_k^{\mu'} \geqslant 0.$$

In the optimum plan the structural limitation is satisfied in the form of an equality. In fact, in the optimum plan let

$$\exists k: l_k^{\mu} + \tilde{l}_k^{\mu} - \alpha_{\mu\mu'} l_k^{\mu'} - \tilde{l}_k^{\mu'} > 0.$$

We reduce l_k^{μ} by δ. Since l_k^{μ} comes into other constraints with a minus sign, the new plan does not depart from the field of permissible solutions, while the objective function, at the same time, is not increased, i.e. the constraint on structure is always satisfied in the optimum plan as an equality.

In contrast to the model of the spatial structure of a region's system of complexes, another additional constraint arises in the model of a complex's spatial structure: namely, a constraint of the forming of a town. By this is meant, at complex level, not the concrete district planning (or layout) of a town but the founding of a settlement point with a certain population in connection with the location of industry. The constraint of the forming of a town is introduced so as to allow for the non-linear character of the inputs on providing for the population connected with the place's growth. The constraint has the following form:

$$\sum_r l_k^r - \gamma \sum_\mu (l_k^\mu + \hat{l}_k^\mu) - \eta \sum_\mu \tilde{l}_k^\mu \geqslant 0 \text{ (shadow prices } \lambda_k);$$

$$-l_k^r \geqslant -L_k^r \text{ (shadow prices } \varphi_k^r).$$

An increase in the number of factors is thus observed, and a detailing of the constraints affecting the location of industry, when territorial systems of lower level are considered. But this pattern does not always apply. The processes of internal migration at complex level, for instance, do not have any essential effect on the location of industry. The territorial elements of a complex are sub-areals, made up (in areas of new development) of small inhabited points and hamlets. On the one hand, therefore, differentiation of the inputs on providing services for the population in the various sub-areals plays no role; on the other hand, the complex is too aggregated an area to be able to account for those factors that operate in an industrial centre, where the problem arises of choosing a variant for a settlement near an enterprise or of creating regular transport services to bring labour to work from old settlements. That is to say, the problems of migration essentially affecting the location of industry at regional or industrial-centre level have no significance at complex level.

The structure of the shadow prices of the manpower block of a complex's model, and the information contained in them, retain all the features of the shadow prices of this block of the model of a region's spatial structure, with the sole difference that the information obtained at complex level characterises each category of labour, taking account of the shadow price of the structural ratio. The shadow price of the constraint of the forming of a town characterises the inputs on providing services for manpower, taking the population's demands on the infrastructure and local resources into account:

$$\lambda_k = c_k^r + \gamma\pi\alpha_k - \varphi_k^r.$$

At regional level this information is contained in the shadow price of the utilisation of manpower.

The constraint of observing the structural ratio between male and female labour facilitates through-going analysis of the process of drawing labour into production. With infringement of the structural ratio

$$\hat{l}_k^{\mu} - \alpha_{\mu\mu'}\hat{l}_k^{\mu'} \gtrless 0,$$

the shadow price of the employment of one category of labour or another, of which there is a surplus, is increased. If, for instance,

$$\hat{l}_k^{\mu} - \alpha_{\mu\mu'}\hat{l}_k^{\mu'} < 0 \text{ then } \tilde{l}_k^{\mu} > 0 \text{ and } \omega_k^{\mu} =$$

$$= \hat{c}_k^{\mu} + \tilde{c}^{\mu'} + \lambda_k\gamma + \eta\lambda_k,$$

i.e. the attraction of labour to complete the structure nearly doubles the shadow price of employing this category of labour. Thus it is unprofitable to draw additional male labour into industry (if there is a surplus of it) because it will require bringing in a certain amount of female labour.

The next block of these regional mesomodels is that of local resources. Investigation of their effect on the location of industry is one of the main points of solving models of this type. With a sectoral or industry approach inputs on the engineering development of an area's land and water resources are treated as a component of the inputs of an enterprise, so that there is no optimisation of the process of exploiting these resources in sectoral models, and the problems of the purpose to which land is put, and its use for industry, farming, and town-building are not considered. This group of problems is reflected in our mesomodels.

The objects treated in the local resources block are water and land, which are characterised by a variant of use and of grading by inputs on development and territorial appurtenance. As regards their spatial relations all these are point objects. In addition, their 'product' is mobile and interchangeable. This can lead to industry's being located in several sub-areals (areals) in the optimum plan, if, of course, there is no need for a whole-number solution. The block consists of equations of two types, as follows:

(1) input-output matrices of the formation of the demand associated with all the preceding blocks;

(2) limitations on the choice of variants for the exploitation of resources (land, water); which limitations are considered for each territorial unit and each variant of the grading of resources.

The constraints have the following form:

$$\sum_{r\in R} S_k^r u_{nk}^r - \sum_{j\in J}\sum_{r\in R} S_{jn}^r x_{jk}^r - \sum_{j\in J}\sum_{r\in R} S_{kn}^{r\varphi} z_k^{r\varphi} - s_n\gamma \sum_{\mu\in M}(l_k^\mu + \hat{l}_k^\mu) \geqslant 0$$

$$(k\in K,\ n\in N); \tag{11.16}$$

$$-\sum_{n\in N} u_{nk}^r \geqslant -1 \ (r\in R,\ k\in K). \tag{11.17}$$

Local resources are investigated in the same detail in regional mesomodels.

The properties of the constraints describing local resources and the information obtained from the shadow prices of this block are similar to those of the manpower block.

The objective function of both models is the minimum summed normalised input on carrying out the given programme by the region (complex). This type of function is most convenient for mesomodels and satisfies the problem of locating production activities and elements of the infrastructure corresponding to the best variant of the use of the territorial industrial combination's resources, given fulfilment of its role in the territorial division of labour.

The formal notations of the standard model takes the following form:

$$\sum_{j\in J}\sum_{r\in R} A_{ij}^{r} x_{jk}^{r} - \sum_{j\in J}\sum_{r\in R}\sum_{\nu\in I} \beta_{i\nu}^{r} A_{\nu j}^{r} x_{jk}^{r} +$$

$$+\sum_{p\in K} x_{ipk} - \sum_{p\in K} x_{ikp} + \hat{x}_{ik} \geqslant B_{ik}\ \ (i\in I;\ (k\in K); \qquad (11.18)$$

$$-\sum_{k\in K}\sum_{r\in R} x_{jk}^{r} \geqslant -1\ \ (j\in J); \qquad (11.19)$$

$$-\hat{x}_{ik} \geqslant -d_{ik}\ \ (i\in I;\ k\in K); \qquad (11.20)$$

$$\sum_{r\in R} \Pi_{kp}^{r\varphi} z_{kp}^{r\varphi} - \sum_{i\in I} \Pi_{i}^{\varphi} x_{ikp} - \sum_{j\in J}\sum_{r\in R} \text{Э}^{r\varphi} x_{jk}^{r} -$$

$$-\sum_{\psi\in\Phi}\sum_{r\in R} \text{Э}_{kr}^{\varphi\psi} z_{k}^{r\varphi} - \pi^{\varphi}\gamma \sum_{\mu\in M} (l_{k}^{\mu} + \hat{l}_{k}^{\mu}) \geqslant 0\ (\varphi\in\Phi;\ p,\ k\in K);$$

$$(11.21)$$

$$-\sum_{r\in R} z_{kp}^{r\varphi} \geqslant -1\ \ (\varphi\in\Phi,\ k,\ p\in K); \qquad (11.22)$$

$$\sum_{r\in R_7} z_{kp}^{r\varphi} - \sum_{r\in R_7} z_{pk}^{r\varphi} \geqslant 0\ \ (\varphi\in\Phi,\ k,\ p\in K); \qquad (11.23)$$

$$\sum_{r\in R} P_{k'}^{r\varphi} z_{k'}^{r\varphi} - \sum_{r\in R}\sum_{k\in K}\sum_{k\in (K-k')} \Pi_{kp}^{r\varphi} z_{kp}^{r\varphi} \geqslant 0\ \ (\varphi\in\Phi;\ k'\in K); \quad (11.24)$$

$$-\sum_{r\in R} z_{k}^{r\varphi} \geqslant -1\ \ (k\in K,\ \varphi\in\Phi); \qquad (11.25)$$

$$l_{k}^{\mu} + \hat{l}_{k}^{\mu} - \sum_{r\in R}\sum_{j\in J} T_{j\mu}^{r} x_{jk}^{r} - \sum_{r\in R}\sum_{\varphi\in\Phi} T_{k\mu}^{r\varphi} z_{k}^{r\varphi} \geqslant 0\ \ (k\in K,\ \mu\in M);$$

$$(11.26)$$

$$-l_{k}^{\mu} \geqslant -T_{k}^{\mu}\,(k\in K,\ \mu\in M); \qquad (11.27)$$

$$\hat{l}_{k}^{\mu} - \alpha_{\mu\mu'} \hat{l}_{k}^{\mu'} + \tilde{l}_{k}^{\mu} - \tilde{l}_{k}^{\mu'} \geqslant 0\ \ (k\in K,\ \mu\in M); \quad (11.28)$$

$$\sum_{r\in R} S_k^r u_{nk}^r - \sum_{j\in J}\sum_{r\in R} S_{jn}^r x_{jk}^r - \sum_{\varphi\in\Phi}\sum_{r\in R} S_{kn}^{r\varphi} z_k^{r\varphi} - s_n\gamma \sum_{\mu\in M}(l_k^\mu + \hat{l}_k^\mu) \geqslant 0$$

$$(k\in K,\ n\in N); \tag{11.29}$$

$$-\sum_{n\in N} u_{nk}^r \geqslant -1\ (r\in R,\ k\in K); \tag{11.30}$$

$$\Big[\sum_{j,k,r} c_{jk}^r x_{jk}^r + \sum_{i,k} \hat{c}_{ik}\hat{x}_{ik} + \sum_{i,p,k} c_{ipk}x_{ipk} +$$

$$+ \sum_{k,p,r,\varphi} c_{kp}^{r\varphi} z_{kp}^{r\varphi} + \sum_{k,r,\varphi} c_k^{r\varphi} z_h^{r\varphi} + \sum_{n,k,r} c_{nk}^r u_{nk}^r +$$

$$+ \sum_{k,\mu} \hat{c}_k^\mu \hat{l}_k^\mu + \sum_{k,\mu} c_k^\mu l_k^\mu + \sum_{k,\mu} \tilde{c}_k^\mu \tilde{l}_k^\mu\Big] \rightarrow \min. \tag{11.31}$$

The models for determining the spatial structure of a region's system of complexes and of an individual complex may thus be distinguished as a special territorial industrial type of regional mesomodel by their place and functions in the system of models and by their specific properties.

ECONOMETRIC ANALYSIS OF TERRITORIAL INDUSTRIAL REGIONAL MESOMODELS

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Territorial industrial mesomodels are tools for pre-planning investigation of the territorial structure of a region's economy. The main job for which they are used is that of optimising the location of production activities within a region. The conception adopted in modelling assumes accounting for the complex effect of regional factors (local resources, infrastructure), and the delineation of the region's main territorial industrial combinations together with their location. In that connection it is very important, for this type of model, to analyse how far the production block of the model is affected by the regional conditions. For that purpose the simplest variant of a mesomodel was adopted here: namely, that of the spatial structure of a region's system of complexes.

The model studied was a static optimising one for long-term planning. The set of all the industries (products) I $(i, q \in I)$ was divided into four sub-sets, as follows:

I_1—the sub-set of products made in the region but not consumed in it, for which there are assignments for inter-regional deliveries;

I_2—the sub-set of products made and consumed in the region, for which there are assignments for inter-regional deliveries;

I_3—the sub-set of products both made in the region and brought in from outside for consumption within it;

I_4—the sub-set of products imported into the region for consumption within it.

$$I_\alpha \cap I_\beta = \varnothing, \quad \alpha \neq \beta; \ \alpha, \ \beta = 1, 2, 3, 4.$$

Inter-regional deliveries are treated as known in the form of assignments for shipping output of the specialisation industries and of limitations on imports into the region.

The set of areals of concentration of production activities K $(k, p, l \in K)$ is broken down into exterior areals (sub-set K_1) and interior areals (sub-set K_2).

UNKNOWNS

x_{ik} —rate of functioning of a production activity making product i and located in areal k $(i \in I;\ k \in K;\ x_{ik} \geqslant 0)$;

z_{ipl} —volume of freights of product i from areal p to areal l $(i \in I;\ p,\ l \in K;\ z_{ipl} \geqslant 0)$;

$\bar{x}_{ik}$ —volume of imports of product i into areal k from outside the region $(i \in I;\ k \in K;\ \bar{x}_{ik} \geqslant 0)$;

y_{nk}^{ind}—amount of land of grade n in areal k used for industrial development ($k \in K$; $n \in N$; N is the set of variants of the gradation of the cost of developing parcels of land; $y_{nk}^{\text{ind}} \geqslant 0$);

y_{nk}^{civ}—amount of land of grade n in areal k used for civil building $(k \in K;\ n \in N;\ y_{nk}^{\text{civ}} \geqslant 0)$;

x_k—amount of labour available in areal k for the production activity concerned $(k \in K;\ x_k \geqslant 0)$;

$\bar{x}_k$—amount of labour brought into areal k $(k \in K;\ \bar{x}_k \geqslant 0)$;

KNOWN (GIVEN) QUANTITIES

A_i—volume of output of product i by a standard enterprise ($i \in I$; $A_i = 0$, when $i \in I_4$; $A_i > 0$ in other cases);

β_{iq}—technological coefficient of the inputs of product i to make a unit of product q in a standard enterprise ($i,\ q \in I$; $\beta_{iq} = 0$, when $i \in I_1 \vee q \in I_4$; $\beta_{iq} > 0$ in other cases);

S_i—area needed to site a standard enterprise to make product i ($i \in I$; $S_i = 0$, when $i \in I_4$; $S_i > 0$ in other cases);

s—unit expenditure of land to provide services for the population $(s > 0)$;

T_i—amount of labour needed to operate a standard enterprise to make product i ($i \in I$; $T_i = 0$, when $i \in I_4$, and $T_i > 0$ in other cases);

B_{ik}—target for shipments of product i through areal k ($i \in I$; $k \in K$; $B_{ik} > 0$ when $i \in I_1 \cup I_2 \wedge k \in K_1$; $B_{ik} = 0$ in other cases);

d_{ik}—limit on bringing product i in from outside the region through areal k ($i \in I$; $k \in K$; $d_{ik} > 0$, when $i \in I_3 \cup I_4 \wedge k \in K_1$; $d_{ik} = 0$ in other cases);

S_{nk} —amount of land of grade n in areal k ($k \in K$; $n \in N$; $S_{nk} > 0$);

$\hat{T}_k$ —manpower reserves of areal k ($k \in K$; $\hat{T}_k > 0$);

c_{ik}—input connected with the building and functioning of a standard enterprise to make product i in areal k ($k \in K$; $i \in I$; $c_{ik} = 0$, when $I \in I_4$; $c_{ik} > 0$ in other cases);

f_{ik} —inputs connected with the building and functioning of an enterprise to make product i outside the region and deliver it through areal k ($k \in K$; $i \in I$; $f_{ik} > 0$, when $i \in I_3 \cup I_4 \wedge k \in K_1$; $f_{ik} = 0$ in other cases);

g_{ipl}—inputs connected with transporting product i from areal p to areal l ($i \in I$; p, $l \in K$; $g_{ipl} > 0$);

c_k—inputs connected with providing services for the population in areal k ($k \in K$; $c_k > 0$);

$\bar{c}_k$—inputs connected with attracting and settling population in areal k ($k \in K$; $\bar{c}_k > 0$);

$\lambda_{nk}^{\text{civ}}, \lambda_{nk}^{\text{ind}}$—inputs connected with preparing grade n land for civil (civ) or industrial (ind) building in areal k ($k \in K$; $n \in N$; $\lambda_{nk}^{\text{civ}} > 0$; $\lambda_{nk}^{\text{ind}} > 0$);

The model contains the following constraints.

The production and distribution of products made in the region are balanced by areals, and deliveries of products of the specialisation industries must be met:

$$A_i x_{ik} + \sum_p z_{ipk} - \sum_l z_{ikl} - \sum_q \beta_{iq} A_q x_{qk} + \bar{x}_{ik} \geqslant B_{ik}$$

(shadow prices u_{ik}). (12.1)

There is a limitation on the possible bringing in of scarce products from outside the region:

$$-\bar{x}_{ik} \geqslant -d_{ik} \ (i \in I, \ k \in K)$$

(shadow prices η_{ik}). (12.2)

Land within an areal is allocated as follows:
(i) to provide sites for industrial facilities

$$\sum_n y_{nk}^{\text{ind}} - \sum_i S_i x_{ik} \geqslant 0 \ (k \in K)$$

$$(\text{shadow prices } \alpha_k^{\text{ind}}); \quad (12.3)$$

(ii) to provide sites for civil construction

$$\sum_n y_{nk}^{\text{civ}} - s(x_k + \bar{x}_k) \geqslant 0 \ (k \in K)$$

$$(\text{shadow prices } \alpha_k^{\text{civ}}); \quad (12.4)$$

(iii) grade n land is limited by areals

$$-y_{nk}^{\text{ind}} - y_{nk}^{\text{civ}} \geqslant -S_{nk} \ (k \in K, \ n \in N)$$

$$(\text{shadow prices } \mu_{nk}); \quad (12.5)$$

Production activities are provided with labour

$$x_k + \bar{x}_k - \sum_i T_i x_{ik} \geqslant 0 \ (k \in K)$$

$$(\text{shadow prices } \omega_k); \quad (12.6)$$

$$-x_k \geqslant -\hat{T}_k \ (k \in K)$$

$$(\text{shadow prices } \theta_k); \quad (12.7)$$

The objective function reflects the summed total inputs on meeting the targets of the region's programme:

$$\min \Big[\sum_{i,k} c_{ik} x_{ik} + \sum_{i,k} f_{ik} \bar{x}_{ik} + \sum_{i,p,l} g_{ipl} z_{ipl} + \sum_k c_k x_k + + \sum_k \bar{c}_k \bar{x}_k + \sum_{n,k} \lambda_{nk}^{\text{ind}} y_{nk}^{\text{ind}} + \sum_{n,k} \lambda_{nk}^{\text{civ}} y_{nk}^{\text{civ}} \Big]. \quad (12.8)$$

As follows from these constraints, the model is built on the input-output principle. The matrix has the following structure:

	x_{ik}		x_k	$\bar{x}_k$	y_{nk}^{ind}	y_{nk}^{civ}	z_{ipl}
I_1	$A_i\ 0\ 0\ 0$						$1\quad -1$
I_2	$-\beta\ A_i\ -\beta\ 0$						$1\quad -1$
I_3	$-\beta\ -\beta\ A_i\ 0$						$1\quad -1$
I_4	$-\beta\ -\beta\ -\beta\ 1$						$1\quad -1$
I_1		$A_i\ 0\ 0\ 0$					$-1\quad 1$
I_2		$-\beta\ A_i\ -\beta\ 0$					$-1\quad 1$
I_3		$-\beta\ -\beta\ A_i\ 0$					$-1\quad 1$
I_4		$-\beta\ -\beta\ -\beta\ 1$					$-1\quad 1$
	$-T_i$		1	1			
		$-T_i$	1	1			
			-1				
			-1				
	$-S_i$				1		
		$-S_i$			1		
			$-s$	$-s$		1	
			$-s$	$-s$		1	
					-1	-1	
					-1	-1	

The matrix $\{\beta_{iq}\}$ is productive for $i,\ q \in I_2 \cup I_3$.

The dual of the primal solution has the following form:

$$A_i u_{ik} - \sum_q \beta_{qi} A_i u_{qk} - \alpha_k^{\text{ind}} S_i - \omega_k T_i \leqslant c_{ik} \quad (i \in I,\ k \in K)$$

(shadow prices x_{ik}); (12.9)

$u_{ik} - \eta_{ik} \leqslant f_{ik}$ $(i \in I;\ k \in K)$ (shadow prices $\bar{x}_{ik}$); (12.10)

$u_{il} - u_{ip} \leqslant g_{ipl}$ $(i \in I;\ p,\ l \in K)$ (shadow prices z_{ipl}); (12.11)

$$\alpha_k^{\text{ind}} - \mu_{nk} \leqslant \lambda_{nk}^{\text{ind}} \quad (k \in K,\ n \in N) \quad \text{(shadow prices } y_{nk}^{\text{ind}}); \tag{12.12}$$

$$\alpha_k^{\text{civ}} - \mu_{nk} \leqslant \lambda_{nk}^{\text{civ}} \quad (k \in K,\ n \in N) \quad \text{(shadow prices } y_{nk}^{\text{civ}}); \tag{12.13}$$

$$\omega_k - s\alpha_k^{\text{civ}} - \theta_k \leqslant c_k \quad (k \in K) \quad \text{(shadow prices } x_k); \tag{12.14}$$

$$\omega_k - s\alpha_k^{\text{civ}} \leqslant \bar{c}_k \quad (k \in K) \quad \text{(shadow prices } \bar{x}_k); \tag{12.15}$$

$$\max \Big[\sum_{i,k} B_{ik} u_{ik} - \sum_{i,k} \eta_{ik} d_{ik} - \sum_{n,k} \mu_{nk} S_{nk} - \sum_k \theta_k \hat{T}_k \Big]. \tag{12.16}$$

Let us consider the conditions for the existence of a solution of the primal system of inequalities.

Assuming that system (12.1) and (12.2) has a solution, we find the ratio between the vector of the constraints and the coefficients of the matrix.

Adding inequalities (12.3) and (12.4) together and integrating for k, we obtain

$$\sum_{n,k} y_{nk}^{\text{ind}} + \sum_{n,k} y_{nk}^{\text{civ}} \geqslant \sum_{k,i} S_i x_{ik} + \sum_k s\,(x_k + \bar{x}_k),$$

which, in place of (12.5), yields:

$$\sum_{i,k} S_i x_{ik} + \sum_k s\,(x_k + \bar{x}_k) \leqslant \sum_{n,k} S_{nk}.$$

Making use of inequality (12.6) we find the ratio between the region's output and local resources (land and water):

$$\sum_{i,k} (S_i + sT_i)\, x_{ik} \leqslant \sum_{n,k} S_{nk}.$$

The proportions between scarce resources (products) brought in from outside the region and their consumption by industry within it are obtained from constraints (12.1) and (12.2). We integrate inequalities (12.1) by k for $i \in I_4$:

$$\sum_k \bar{x}_{ik} - \sum_{q,k} \beta_{iq} A_q x_{qk} \geqslant 0 \quad (B_{ik} \text{ for } i \in I_4 \text{ is equal to } 0)$$

and, since it follows from (12.3) that

$$\sum_k \bar{x}_{ik} \leqslant \sum_k d_{ik},$$

then the constraint sought has the form

$$\sum_{q,k} \beta_{iq} A_q x_{qk} \leqslant \sum_k d_{ik}.$$

It will be readily seen that the ratio between the region's output and manpower does not fall within the field of the permitted solution, since there is no bilateral limitation on the variables $\bar{x}_k$ (the amount of labour attracted from outside the region).

We consider the relationship between the output of products of various groups* and the targets for shipping output of the specialisation industries out of the region.

Having integrated inequalities (12.1) for k, we eliminate variables z_{ipl}. For $i \in I_1$ constraint (12.1) takes the form:

$$\sum_k A_i x_{ik} \geqslant \sum_k B_{ik} \Rightarrow \sum_k x_{ik} \geqslant \frac{\sum\limits_k B_{ik}}{A_i}.$$

For $i \in I_2 \cup I_3$ constraint (12.1) is transformed into the following:

$$\sum_k A_i x_{ik} - \sum_{q,k} \beta_{iq} A_q x_{qk} + \sum_k d_{ik} \geqslant \sum_k B_{ik}.$$

From $\sum\limits_{q,k} \beta_{iq} A_q x_{qk}$ we isolate the sum reflecting the demand of the production activities of the specialisation industries ($q \in I_1$), and substitute $\sum\limits_k x_{qk}$ for $q \in I_1$ in $\sum\limits_k \frac{B_{qk}}{A_q}$. Since $\sum\limits_k \frac{B_{qk}}{A_q} \leqslant \sum\limits_k x_{qk}$, the sign of the inequality is not altered, while constraint (12.1) is transformed for $i \in I_2 \cup I_3$ into:

$$\sum_k A_i x_{ik} - \sum_k \sum_{q \in I_2 \cup I_3} \beta_{iq} A_q x_{qk} + \sum_k d_{ik} \geqslant \sum_k B_{ik} + \sum_{k, q \in I_1} \beta_{iq} B_{qk},$$

or

$$\sum_{q \in I_2 \cup I_3} (\delta_{iq} - \beta_{iq}) A_q \sum_k x_{qk} \geqslant \sum_k B_{ik} + \sum_{k, q \in I_1} \beta_{iq} B_{qk} - \sum_k d_{ik}.$$

We denote:

$$\sum_k B_{ik} + \sum_{k, q \in I_1} \beta_{iq} B_{qk} - \sum_k d_{ik} = D_i,$$

---

* Product groups correspond to sub-set I_α of set I.

then the ratio obtained can be written as a vector:

$$(E-\beta)\,\overline{A}\,\overline{X} \geqslant \overline{D},$$

in which $\overline{A}$ is the diagonal matrix with elements A_i along the diagonal:

$$\begin{pmatrix} A_1 & 0 & \dots & 0 \\ \cdot & A_2 & & \cdot \\ \cdot & & \ddots & \cdot \\ 0 & \dots & \dots & A_n \end{pmatrix}$$

$\overline{X}$ is the vector with components $\sum_k x_{qk}$:

$$\left(\sum_k x_{1k},\ \sum_k x_{2k},\ \dots,\ \sum_k x_{nk}\right).$$

The matrix $\{\beta_{iq}\}$ for $i \in I_2 \cup I_3$ is productive; consequently, there is an opposite for $(E-\beta)$ with non-negative elements. In consequence of that, multiplication of both parts of the inequality by $(E-\beta)^{-1}$ does not disturb the order of the terms introduced into the set of $\overline{X}$ vectors:

$$\overline{A}\,\overline{X} \geqslant (E-\beta)^{-1}\,\overline{D}.$$

We multiply both terms of the inequality by diagonal matrix M, of the form

$$\begin{pmatrix} \frac{1}{A_1} & 0 & \dots & 0 \\ \cdot & \frac{1}{A_2} & & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ 0 & \dots & & \frac{1}{A_n} \end{pmatrix}$$

and find that vector $\overline{X}$ satisfies the relationship

$$\overline{X} \geqslant M\,(E-\beta)^{-1}\,\overline{D}.$$

We denote the matrix as $M\,(E-\beta)^{-1}D = H$, then $\sum x_{ik} \geqslant H_i$. Thus, through the coefficients of material inputs and targets for deliveries of the output of the specialisation industries, the lower limits for volumes of output $i \in I_2 \cup I_3$ are obtained.

Inserting the limits of $\sum_k x_{ik}$ found for $i \in I_1$ and $i \in \in I_2 \cup I_3$ in the relationship linking output with local and outside resources, we obtain the necessary conditions for the existence of an admissible solution reflecting the dependence between the vector of the constraints and the coefficients of the matrix:
(i) for local resources

$$\sum_{i\in I_1} (S_i+sT_i) \sum_k \frac{B_{ik}}{A_i} + \sum_{i\in I_2 \cup I_3} (S_i+sT_i)\, H_i \leqslant \sum_{n,\, k} S_{nk}; \tag{12.17}$$

and (ii) for resources brought in from outside the region

$$\sum_{q\in I_1} \beta_{iq} \sum_k B_{ik} + \sum_{q,\, k} \beta_{iq} A_q H_q \leqslant \sum_k d_{ik}. \tag{12.18}$$

Constraints (12.17) and (12.18) have a simple economic sense: namely, the region must have adequate resources to meet the delivery targets for output of the specialisation industries, taking the region's comprehensive development into account.

The relationships (12.17) and (12.18) are adequate for there to be an admissible solution.

Owing to the limited nature of the set of admissible solutions, the primal problem has an optimum solution.

For models of the location of industry it is extremely important that there are no cross and cyclic freights in the optimum plan. Let us see whether our model meets this requirement.

Property 1. There are no cross and cyclic freights in the optimum plan.

Proof. Suppose there to be even one cyclic freight in the optimum plan. We reduce all z_{ikp} entering the ε cycle. Since, for any peak (areal) of the circuit there is both a variable import and a variable export proper to the circuit, the limitation will not be infringed, but the objective function will be reduced. Thus a plan containing cyclic freights will not be optimal. We go on increasing ε until the cycle is broken. Shuttle freights represent a specific case of the cyclic ones. The statement, consequently, is true.

The model reviewed describes an economic situation such that the region's place in the inter-regional division of labour is known, i.e. its assignment for output of the specialisation industries has been fixed. With that, overfulfilment of the target is not encouraged. This situation is reflected in the following properties.

Property 2. Constraint (12.1) is reflected in the optimum plan in the equality of $\forall i \in I$, $k \in K$. The shadow prices of the fulfilment of delivery targets of the specialisation industries are positive in the optimum plan, i.e. $u_{ik} > 0$ for $\forall i \in I_1 \cup I_2$, $k \in K_1$.

Proof. Suppose there to be a pair $(\hat{i}, \hat{k})$ in the optimum plan for which the following is valid

$$A_{\hat{i}} x_{\hat{i}\hat{k}} + \sum_p z_{\hat{i}p\hat{k}} - \sum_l z_{\hat{i}\hat{k}l} + \bar{x}_{\hat{i}\hat{k}} - \sum_q \beta_{\hat{i}q} A_q x_{q\hat{k}} > B_{\hat{i}\hat{k}}. \tag{12.19}$$

With that, at least one of the constraints is satisfied:

$$z_{\hat{i}p\hat{k}} > 0; \quad \bar{x}_{\hat{i}\hat{k}} > 0; \quad x_{\hat{i}\hat{k}} > 0.$$

In the first two cases we reduce $z_{\hat{i}p\hat{k}}$ $(\bar{x}_{\hat{i}\hat{k}})$ by ε, but do not leave the field of admissible solutions, since $z_{\hat{i}p\hat{k}}$ $(\bar{x}_{\hat{i}\hat{k}})$ comes into the other constraints with a minus sign. The procedure is continued until equality is established in (12.19).

Let $x_{\hat{i}\hat{k}}$ be greater than zero; then by the condition of complementary non-rigidity, constraint (12.9) is satisfied for $(\hat{j}, \hat{k})$ in the form of an equality

$$A_{\hat{i}} u_{\hat{i}\hat{k}} - \sum_q \beta_{q\hat{i}} A_{\hat{i}} u_{q\hat{k}} - \alpha_{\hat{k}}^{\text{ind}} S_{\hat{i}} - \omega_{\hat{k}} T_{\hat{i}} = c_{\hat{i}\hat{k}}.$$

On the other hand, it follows from constraint (12.19) that $u_{\hat{i}\hat{k}} = 0$, i.e. (12.19) has the form

$$-\sum_q \beta_{q\hat{i}} A_{\hat{i}} u_{q\hat{k}} - \alpha_{\hat{k}}^{\text{ind}} S_{\hat{i}} - T_{\hat{i}} \omega_{\hat{k}} = c_{\hat{i}\hat{k}},$$

which is impossible by virtue of the positiveness of $\beta_{q\hat{i}}$, $A_{\hat{i}}$, $S_{\hat{i}}$, $T_{\hat{i}}$, $c_{\hat{i}\hat{k}}$, and the non-negativity of $u_{q\hat{k}}$, $\alpha_{\hat{k}}^{\text{ind}}$, and $\omega_{\hat{k}}$.

Suppose that $\exists$ is the pair $(\hat{i}, \hat{k})$ $i \in I_1 \cup I_2$, $k \in K_1$, for which $u_{\hat{i}\hat{k}} = 0$, then (12.9) has the form

$$-\sum_q \beta_{q\hat{i}} A_{\hat{i}} u_{q\hat{k}} - S_{\hat{i}} \alpha_{\hat{k}}^{\text{ind}} - T_{\hat{i}} \omega_{\hat{k}} < c_{\hat{i}\hat{k}}.$$

By the condition of complementary non-rigidity, $x_{\hat{i}\hat{k}} = 0$, consequently there is a p such that $z_{\hat{i}p\hat{k}} > 0$. Otherwise constraint (12.1) is violated.

With $z_{\hat{i}p\hat{k}} > 0$, constraint (12.11) has the following form, by the condition of complementary non-rigidity:

$$u_{\hat{i}\hat{k}} - u_{\hat{i}p} = g_{\hat{i}p\hat{k}}.$$

According to the proposition, $u_{\hat{i}\hat{k}} = 0$, consequently $-u_{\hat{i}p} = g_{\hat{i}p\hat{k}}$. Thus the proposition that $u_{\hat{i}\hat{k}} = 0$ does not hold for $i \in I_1 \cup I_2$, $k \in K_1$.

Property 3. If $x_{ik} > 0 \vee \bar{x}_{ik} > 0 \vee z_{ipk} > 0$ for $i \in I_3 \cup I_4$, $k \in K_1$ or $i \in I$, $k \in K_2$, then $u_{ik} > 0$.

If $u_{ik} = 0$ for (i, k): $i \in I_3 \cup I_4$, $k \in K_1$ or $i \in I$, $k \in K_2$, then $x_{ik} = 0 \wedge \bar{x}_{ik} = 0 \wedge z_{ipk} = 0 \wedge z_{ikl} = 0$.

Thus, when areal k is involved in the interareal division of labour as a producer of output or a transfer point, the areal has a shadow price differing from zero as regards the production and distribution of output.

Proof. Let $\bar{x}_{ik} > 0$. By the condition of complementary non-rigidity, constraint (12.10) has the form $u_{ik} - \eta_{ik} = f_{ik}$, and by virtue of the positivity of f_{ik}, u_{ik} is positive. If x_{ik} or z_{ipk} is greater than zero, the proof of x_{ik}, z_{ipk} is similar.

With $u_{ik} = 0$, constraint (12.9) has the form

$$-\sum_{q} \beta_{qi} A_i u_{qk} - T_i \omega_k - S_i \alpha_k^{\text{ind}} < c_{ik} \Rightarrow x_{ik} = 0.$$

In accordance with Property 2 constraint (12.1) becomes an equality. Two cases are possible: either all the variables are zero, and then the statement holds; or the sum of the negative terms equals the sum of the positive terms. Consequently, $\bar{x}_{ik} > 0 \vee z_{ipk} > 0$.

If $\bar{x}_{ik}$ is greater than zero, (12.10) becomes an equality by the principle of complementary non-rigidity, and $u_{ik} = 0$. Similarly, if $z_{ipk} > 0$.

As we have already remarked, analysis of the effect of regional factors on the siting of industry is our main interest. Consider constraints (12.3) to (12.7) inclusive, which describe the use of land and manpower.

One of the most important problems of representing manpower in regional models is the method of accounting for migration. In the model being discussed the migration streams are accounted for differently depending on their direction. The possible attraction of labour from outside the region is reflected in variable $\bar{x}_k$, which makes it possible to account for a shortage of manpower by areals and obtain additional information on the forming of the optimum plan for the siting of production activities. The exodus is taken into account through expert estimates of the increase in the coefficient of production activities' demand for labour. The problem of reflecting migration within a region in the model is still unresolved.

Let us consider the properties to which the limitations on labour and certain patterns of the formation of their shadow prices are subordinated in the optimum plan.

Property 4. Constraint (12.6) is satisfied in the optimum plan as an equality.

In the optimum plan it holds that, if $\exists\, i\colon x_{ik} > 0$, then $\omega_k > 0$, and if $\omega_k = 0$, then $x_{ik} = 0$ for $\forall i$. That is to say, if the areal is involved in the interareal division of labour for the production of even one item, the utilisation of its manpower has a shadow price other than zero.

If an areal has a zero shadow price for the employment of labour, no industry is located in it.

Proof. Suppose that there is a k for which $x_k + \bar{x}_k - \sum_i T_i x_{ik} > 0$. We reduce x_k (or $\bar{x}_k$, if $\bar{x}_k \neq 0$) by ε. It will be seen that the new plan is within the range of admissible solutions. The process is continued (i.e. ε is increased) until there is an equality in (12.6).

Suppose that there is an i for which $x_{ik} \neq 0$. Then, at least, $x_k > 0$. By the condition of complementary non-rigidity, $\omega_k - s\alpha_k^{\text{civ}} - \theta_k = c_k$. Consequently, $\omega_k > 0$.

Let $\omega_k = 0 \Rightarrow -s\alpha_h^{\text{civ}} - \theta_k < c_k$ and $-s\alpha_k^{\text{civ}} < \bar{c}_k$. By the condition of complementary non-rigidity $\bar{x}_k = 0 \wedge x_k = 0 \Rightarrow -\sum_i T_i x_{ik} \geqslant 0$, which means that $x_{ik} = 0$ for $\forall i$.

An important step in the mathematical economic analysis of the model is to clarify the structure of the shadow prices.

From the economic formulation of the problem of specialisation of the system of complexes within a region it follows that not only must the limited character of the regional factors be taken into account, but also certain qualitative changes in the character of their effect on the location of industry at the given stage of planning must be allowed for. (These changes should be expressed, in our view, not by increasing the number of regional factors considered but in a stronger, more comprehensive effect compared with the regional economic models.) Let us consider how this requirement is reflected in the models being studied.

In the optimum plan $x_k > 0$, if the areal is involved in the production of even one product, since the inputs on attracting labour from outside the region and providing it with housing and services are greater than the inputs on local manpower. From the positivity of x_k it follows, by the condition of complementary non-rigidity, that $\omega_k = = c_k + s\alpha_k^{\text{civ}} + \theta_k$. The shadow price of utilising manpower in the areal exceeds the shadow prices of the quantity of labour in the areal by $c_k + s\alpha_k^{\text{civ}}$, i.e. it characterises not only the quantity of labour but also the inputs on providing it with services, including the areal's shadow price of providing workers and their families with land for dwellings. It can be shown that with the limitations on electricity and water considered in the model, the shadow price of utilising labour in the areal includes the normalised inputs on its employment, the shadow price of the quantity of labour in the areal, and the sum of the shadow prices of the areal for the utilisation of all regional conditions for the needs of the population. This conclusion is confirmed by practical computations.

Thus the break down of the limitations on manpower into two conditions, namely (a) the input-output matrix of its employment and (b) the limitation fixed at higher level on the variable of the amount of labour employed, makes it possible to give a qualitatively better characteristic of the involvement of the areal in the interareal division of labour.

The economic regions of Siberia are short of labour; it is therefore of interest to consider that case in which labour is drawn from outside the region, i.e. $\exists k : \bar{x}_k > 0$. By the condition of complementary non-rigidity $-s\alpha_k^{\text{civ}} + \omega_k = \bar{c}_k \Rightarrow \omega_k = \bar{c}_k + s\alpha_k^{\text{civ}}$. And because local reserves are also employed, i.e. because $x_k > 0$,

$$\omega_k = c_k + s\alpha_k^{\text{civ}} + \theta_k \Rightarrow \theta_k = \bar{c}_k - c_k.$$

The shadow price of the pool of labour in the areal is the difference between the total inputs on attracting manpower to the areal from outside the region, and the inputs on employing local labour. The higher the inputs on local labour, consequently, the lower is the shadow price of the pool. With canonic interpretation of the shadow price for increasing the amount of labour in the areal by one unit, the value of the objective function is diminished by θ_k, i.e. the higher the inputs on employing local labour, in our case, relative to the inputs on attracting labour, the lower the efficiency of the areal's manpower will be compared with other areals.

Let us consider in more detail the problem of forming the constraints that describe the functioning of manpower in an areal. As already mentioned, the constraint on manpower in the model being studied is divided into the input-output matrix of the employment of labour and the limitation from above on the variable of the quantity of local labour employed [see constraints (12.6) and (12.7)].

Another formalisation is possible, without distinguishing the variable x_k. It can easily be shown that, with a certain method of accounting for input (in conditions of labour shortage), the shadow prices of the siting of the same types of output coincide in both cases. Thus both ways of accounting for labour are legitimate, but with the second one the dimension of the problems is reduced. This singling out of a variable of the use of local labour, however, makes it possible (1) to allow for the effect of local factors on the location of industry, and (2) to take account of their effect as well with partial utilisation. In the light of these reasons, the formalisation introduced into the model under review is preferable, in our view.

The formalisation of the use of land and its properties are similar to the formalisation and properties of manpower. In fact inequalities (12.3) and (12.4), which reflect the allocation of land in the model are subordinate to Property 5, the sense and proof of which are similar to those of Property 4 characterising the formalisation of manpower.

Property 5. Constraints (12.3) and (12.4) are satisfied in the optimum plan as equalities.

In the optimum plan it holds that, if $\exists i : x_{i\hat{k}} > 0$, then $\alpha_{\hat{k}}^{\text{ind}} > 0 \wedge \alpha_{\hat{k}}^{\text{civ}} > 0$; and if $\alpha_{\hat{k}}^{\text{ind}} = 0 \vee \alpha_{\hat{k}} = 0$, then $x_{ik} = 0$ for $\forall i$.

Consider the structure of the shadow prices of the limitations on land. If the areal is involved in the territorial division of labour as a producer of output, i.e. $\exists i : x_{ik} > 0$, then $y_{nk}^{\text{ind}} > 0$, and $y_{nk}^{\text{civ}} > 0$, i.e. land is used for both industrial and civil building in the areal. By the condition of complementary non-rigidity, limitations (12.14) and (12.15) of the dual solution become equalities: $\alpha_k^{\text{ind}} - \mu_{nk} = \lambda_{nk}^{\text{ind}}$;

$$\alpha_k^{\text{civ}} - \mu_{nk} = \lambda_{nk}^{\text{civ}} \Rightarrow \mu_{nk} = \lambda_{nk}^{\text{ind}} - \alpha_k^{\text{ind}} = \lambda_{nk}^{\text{civ}} - \alpha_k^{\text{civ}}. \qquad (12.20)$$

We find that β_{nk}, i.e. the shadow price of land of grade n (cost of development) in areal k, is the difference between the shadow price of the direction of using land and the total inputs on developing it, i.e. the difference between the shadow price of the direction of using land and the total inputs on developing it by the given variant of utilisation, is identical for all variants of use and equal to the shadow price of the variant of the land's grading.

It follows from (12.20) that $\alpha_k^{\text{ind}} - \alpha_k^{\text{civ}} = \lambda_{nk}^{\text{ind}} - \lambda_{nk}^{\text{civ}}$, i.e. that the difference between the shadow prices of the variants of the use of land is the difference between the normalised inputs on developing it by the corresponding variants of use. This difference is constant for all grades of land.

The model of the spatial structure of a region's system of complexes thus satisfied one of the most important requirements of regional mesomodels, i.e. it allows for the comprehensive effect of local factors on the location of industry. In fact, the formalisation of regional conditions employed in the model makes it possible to allow (1) for their complex effect on the shaping of the optimum plan, which corresponds to the requirement of the given level of planning, and (2) for their effect as well with partial utilisation of the factors. The structure of the shadow prices of the limitations describing the use of regional factors reflects the method of accounting for the inputs adopted in constructing the model.

In the case of full exploitation of local resources the dimension of the model can be reduced. This is particularly important when problems are being solved for the labour-hungry regions of Siberia.

The condition of the existence of a solution makes it possible to establish the consistency of the initial information. An important property of the model is that there are no cross or cyclic freights in the optimum plan.

The fact that the structure of regional mesomodels makes it possible to investigate them by the simple, efficient apparatus of duality theory should be noted in particular; thanks to that we can get information on the structure of the shadow prices of the optimum plan, establish certain properties of the model, and see whether the problem formulated can be solved by this model.

13

INTERPRETATION OF THE SHADOW PRICES OF SPATIAL MODELS OF TERRITORIAL INDUSTRIAL COMPLEXES

N. I. Larina

The spatial models of territorial industrial complexes, developed and being used at present in the USSR, belong to the class of linear programming models by which both primal and dual solutions are obtained, as a result of which shadow prices can be obtained. One of the main properties of shadow prices is their concreteness, thanks to which they are widely used in mathematical economic analysis of the solutions of economic problems. We propose employing them to form the information links between the models of our system.

As already noted, the models of complexes form a special type of econometric model that differs from those of separate sectors of the economy and intersectoral programme complexes, models of the point economy, and national economic spatial models. In view of that the shadow prices of the spatial models of complexes will have their own economic character proper only to models of this type. Three groups of shadow prices can be distinguished in them:

(1) shadow prices of the targets for shipments of the output of the specialisation industries, or of the assignments for siting enterprises;

(2) shadow prices of the input-output matrices of the production and distribution of products (resources, services) in the various parts of a region's territory (areals, sub-areals);

(3) shadow prices of the limitations on the scale of production of output (services) for the different variants of the functioning of enterprises of the specialisation and complementary industries and facilities of the infrastructure in the various areals (sub-areals); shadow prices of the limitations on the scale of utilisation of unused capacities of previously built facilities of the infrastructure; shadow prices of the limitations on the scale of exploitation of various categories of national and local resources.

Shadow prices of the first group indicate the magnitude and structure of the full national inputs on meeting targets, given proportional development of the elements concerned of the economy of the area being studied, and rational exploitation of its resources. The shadow prices of the second group indicate the size and structure of the unit outlays of any consumer of output of the specialisation and complementary industries, of the product (services) of the infrastructure, and of national and local resources at the sites of production activities of the infrastructure, and in settlements. And, finally, the shadow prices of the third group indicate the efficiency of producing output (services) by different variants in separate areals (sub-areals), of using underutilised capacities of previously built facilities of the infrastructure, and of exploiting resources of various category. These shadow prices indicate the rent or profit that the region (complex) or separate industries will benefit, if all the consumers considered bear outlays for using output (services, resources) at the shadow prices of the second group.

The structure and economic sense of the shadow prices of each group, and the link between them, can be brought out and established by analysing the conjugate constraints of the primal and dual solutions. The model of the spatial structure of a region's system of complexes considered above (in Chapter 4), and the model of a complex's spatial structure (Chapter 6), are very unwieldy and cumbersome for such analysis. The interconnections in them between the main facilities of the objects of investigation and between the different elements of these objects are described in detail in these models, which bring out the specific features of the economic problems solved by means of them, but complicate analysis of the shadow prices. Here we shall consider simplified models, in which the character of the relations between the main objects of investigation is preserved, while the interconnections between their separate elements are represented in summed or aggregate form. To write these simplified models we employ a new notation that helps avoid the need to indicate the field of change of the models' (sets') parameters, and so simplifies the structure of the expressions considered.

1. SHADOW PRICES OF A SIMPLIFIED MODEL OF THE SPATIAL STRUCTURE OF A REGION'S SYSTEM OF COMPLEXES

The mathematical writing of the model under consideration is given in Diagram 13.1. In contrast to the full model, not only are the volumes and directions of shipment in and out of output indicated, but also the areas of shipment, which makes it possible to obtain shadow prices of the targets for the supping out of output and of the limit on the bringing in of products by areas. The constraints 1 and 3 on the production and distribution of output of the specialisation industries in the simplified model (in Diagram 13.1) correspond to constraints (4.1) and (4.2) of the full model; constraints 2 and 4 of the simplified model (Diagram 13.1) correspond to constraints (4.3) and (4.5) of the full model. The constraints on the building and functioning of the infrastructure are represented in aggregated form in the simplified model; constraints 7 and 8 on the functioning of the transport system in the simplified model correspond to constraints (4.10) to (4.12) inclusive of the full model; constraints 14 and 15 on the functioning of the power system in the simplified model correspond to constraints (4.22) to (4.30) inclusive of the full model. The building industry is not considered in the simplified model. The allocation of areals' local resources (land, water), is aggregated and indicated without distinguishing types, i.e. constraints 14 and 15 correspond to constraints (4.15) to (4.21) inclusive.

As in the full model, the conditions for the use of local and imported manpower in areals is indicated, but a limitation on the number of jobs in the region as a whole is introduced, which allows us to consider variants of the building and siting of production activities, and to differentiate them, in addition to other characteristics, by indices of the employment of manpower and capital investment, and to obtain shadow prices of the limit on manpower for the region. The optimality criterion has the same economic nature as in the full model.

The economic sense and structure of the shadow prices are readily brought out by analysing the correlation of the constraints of the primal and dual solutions for the objects of investigation. In the simplified model the specialisation industries are taken as these objects, plus the complementary industries, the region's transport and power systems, and areals' manpower and other local resources.

Let us consider the concrete sense of the shadow prices of the constraints on the functioning of these elements of the economy in examples representing different situations of the optimum plan.*

Shadow prices of the specialisation industries. These are the shadow prices (a) of the use of output (w_{ik}), (b) of the efficiency of production (β_{ik}), and (c) targets for out-shipments of the product ($u_{i\bar{k}}$). The economic sense can be brought out by analysing equations 1, 3, and 6 of the primal solution (Diagram 13.1) and the first, and fourth to sixth dual solutions (Diagram 13.2).

Situation 1. Let product i be made in areals k_1 and k_2 and shipped out through boundary areal k_3 to region $\bar{k}$. The target for shipments of product i to region $\bar{k}$ is thus met, the production capacity of variant r_1 of an enterprise located in areal k_1 is fully utilised, but that of variant r_2 of an enterprise located in areal k_2 is not fully used.

By considering the corresponding equations of the primal and dual solutions, we write:

$$x^{r_1}_{ik_1} = 1 \Rightarrow \beta_{ik_1} \geqslant 0;\ \ w_{ik_1} = \frac{S^{r_1}_{ik_1}}{A^{r_1}_{ik_1}} +$$

$$+ \sum_j a^{r_1}_{ij} w_{jk_1} + \sum_p a^{r_1}_{pi} w_{pk_1} + a^{r_1}_{\text{э}i} w_{\text{э}k_1} + a^{r_1}_{\xi i} w_{\xi k_1} +$$

$$+ \sum_\nu a^{r_1}_{\nu i} w_{\nu k_1} + \frac{\beta_{ik_1}}{A^{r_1}_{ik}} = \text{З}^{r_1}_{ik_1} + \frac{\beta_{ik_1}}{A^{r_1}_{ik_1}}\ ; \qquad (13.1)^{**}$$

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* For models of this type the existence of an optimum plan, the absence of cyclic cross freights, strict observance of the targets for shipping output of the specialisation industries, and the positivity of the shadow prices for the utilisation of output have been demonstrated (see Chapter 12). These properties and the apparatus of the theory of duality (theorem of duality and the condition of complementary non-rigidity) are employed in the rest of the exposition.

** Here, and subsequently, we see the symbol З^r_{ik} to shorten the writing of the different expressions, bearing in mind that

$$\text{З}^r_{ik} = \frac{S^r_{ik}}{A^r_{ik}} + \sum_j a^r_{ji} w_{jk} + \sum_p a^r_{pi} w_{pk} + a^r_{\text{э}i} w_{\text{э}k} + a^r_{\xi i} w_{\xi k} + \sum_\nu a^r_{\nu i} w_{\nu k}.$$

$$0 < x^{r_2}_{ik_2} < 1 \Rightarrow \beta_{ik_2} = 0,\ \ w_{ik_2} = З^{r_2}_{ik_2}; \tag{13.2}$$

$$v_{ik_3\bar{k}} > 0 \Rightarrow u_{i\bar{k}} = w_{ik_3}; \tag{13.3}$$

$$v_{ik_1k_3} > 0 \Rightarrow w_{ik_3} = w_{ik_1} + c_{ik_1k_3} + \alpha_{(k_1k_3)} = \\ = З^{r_1}_{ik_1} + \frac{\beta_{ik_1}}{A^{r_1}_{ik_1}} + c_{ik_1k_3} + \alpha_{(k_1k_3)}; \tag{13.4}$$

$$v_{ik_2k_3} > 0 \Rightarrow w_{ik_3} = w_{ik_2} + c_{ik_2k_3} + \alpha_{(k_2k_3)} = \\ = З^{r_2}_{ik_2} + c_{ik_2k_3} + \alpha_{(k_2k_3)}. \tag{13.5}$$

Comparing the last two equations, we find:

$$\beta_{ik_1} = A^{r_1}_{ik_1}\,[З^{r_2}_{ik_2} - З^{r_1}_{ik_1} + c_{ik_2k_3} + \alpha_{(k_2k_3)} - c_{ik_1k_3} - \alpha_{(k_1k_3)}]. \tag{13.6}$$

Substituting this expression in equation (13.1) we write:

$$w_{ik_1} = З^{r_2}_{ik_2} + [c_{ik_2k_3} + \alpha_{(k_2k_3)} - c_{ik_1k_3} - \alpha_{(k_1k_3)}]. \tag{13.1a}$$

By analysing the structure of these expressions and comparing them, we bring out the specific nature of each of the shadow prices being studied and establish the character of the link between them.

The shadow price of the use of the product (w_{ik}) characterises the outlay of consumers of this product. The size of the shadow price indicates how much the outlay for the region will be increased, if demand for this product in any one areal rises by a unit. The structure of this shadow price indicates what occasions the rise in inputs.

In the limiting areal of a production activity* the shadow price of exploitation is equal to the full unit inputs on its output, i.e. to expression (13.2), which embraces the direct unit input corresponding to the production variant employed in the optimum plan $\left(\frac{S^{r_2}_{ik_2}}{A^{r_2}_{ik_2}}\right)$, and the sum of the unit inputs of the feedback arising through development of elements of the economy in the region serving the given variant.

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* The areal in which a production activity is sited has a zero shadow price of the efficiency of production in the optimum plan. In the situations being considered above, only those cases are analysed in which the capacities of the production activities do not reach the upper limits,

In the model being investigated the feedback inputs consist of the region's total input on producing the output of the specialisation and complementary industries in the volumes consumed in the given production process

$$\left(\sum_{j} a^{r_2}_{ji} w_{jk_2} + \sum_{p} a^{r_2}_{pi} w_{pk_2}\right);$$

the total input on power supply $(a^{r_2}_{\text{э}i} w_{\text{э}k_2})$, the total input on settling and serving the population and developing the service industries and infrastructure $(a^{r_2}_{\xi i} w_{\xi k_2})$ and the full input arising from exploitation of areals' local resources $\left(\sum_{v} a^{r_2}_{vi} w_{vk_2}\right)$.

The given shadow price is represented in the shadow prices (i) of the use of products of other areals, (ii) of the targets for import of products, and (iii) of the efficiency of production, i.e. expressions (13.5) and (13.1a), (13.3), and (13.6). This means that the level of the region's total unit inputs on output in the marginal areal of production affects the outlays of all consumers wherever they are situated, and on the efficiency of the functioning of similar production activities in other areals.

In an areal in which there is no local industry the shadow price of the use of a product is equal to the sum of two quantities: namely, (a) the region's total unit costs of its output in the marginal areal of the production activity; and (b) the size of the total transport costs of delivering a unit of product from the areal of the production activity, i.e. expression (13.5).

In the effective areal of a production activity* the shadow price of the use of a product is determined by the level of the region's total unit costs of its output in the marginal areal of production and the difference in the transport costs of delivering a unit of the product to the limiting region of consumption** from the marginal and effective areals of production, i.e. expression (13.1a). The shadow price of the efficiency of production is equal to the sum of two quantities: namely, (a) the difference between the region's total inputs on producing output in the marginal areal and in the areal concerned; and (b) the difference between the total cost of transporting the product from these areals to the marginal region of consumption, i.e. expression (13.6).

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* The areal in which the production activity is sited has a positive shadow price of efficiency in the optimum plan. In this case the capacity of the activity reaches the upper limits.

** The region (areal) in which demand for the product under consideration is satisfied in whole or in part from the marginal areal of the production activity.

The shadow price of shipping a product out of the region ($u_{i\bar{k}}$) shows what costs the region bears in producing and transporting a unit of the product to the boundary areal. Its value is always determined by the level of the region's total unit cost of making the product in the marginal areal of the production activity and the level of the total transport costs of delivering a unit from this areal to the boundary areal, i.e. expressions (13.3) and (13.5).

The shadow prices of the complementary industries are the shadow prices of the use of a product (w_{pk}), of the efficiency of a production activity (α^r_{pk}), and the efficiency of imports of the product ($\eta_{p\hat{k}}$). In order to bring out the economic nature of these shadow prices, we assume (as in the optimum plan) that equations 2 and 5 of the primal solution (Diagram 13.1) and 2° and 7° to 9° of the dual solution (Diagram 13.2) are met.

Situation 2. Let product p be produced in areals k_1 and k_3 and shipped to areal k_2. Suppose that the optimum plan includes variants r_1 and r_2 of the output of product i in areal k_1 and variant r_3 in areal k_3. Then the productive capacities of the r_1 and r_3 variants are fully used, but those of variant r_2 are underutilised.

By analysing these equations as indicated in Situation 1, we can immediately write the values of the shadow prices being investigated:

$$w_{pk_1} = s^{r_2}_{pk_1} + \sum_i a^{r_2}_{ip} w_{ik_1} + \sum_{p'} a^{r_2}_{p'p} w_{p'k_1} + a^{r_2}_{эp} w_{эk_1} +$$

$$+ a^{r_2}_{\xi p} w_{\xi k_1} + \sum_{\nu} a^{r_2}_{\nu p} w_{\nu k_1} = 3^{r_2}_{pk_1}\,*; \qquad (13.7)$$

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* Here and subsequently in this chapter we use the symbol 3^r_{pk} to write the separate expressions in order to save space: thus $3^r_{pk} = s^r_{pk} + \sum_i a^r_{ip} w_{ik} + \sum_{p'} a^r_{p'p} w_{p'k} + a^r_{эp} w_{эk} + a^r_{\xi k} w_{\xi k} + \sum_{\nu} a^r_{\nu p} w_{\nu k}$.

$$\alpha^{r_1}_{pk_1} = З^{r_2}_{pk_1} - З^{r_1}_{pk_1}, \ \alpha^{r_2}_{pk_1} = 0; \tag{13.8}$$

$$w_{pk_2} = З^{r_2}_{pk_1} + c_{pk_1k_2} + \alpha_{(k_1k_2)}; \tag{13.9}$$

$$w_{pk_3} = З^{r_3}_{pk_3} + \alpha^{r_3}_{pk_3} = З^{r_2}_{pk_1} + [c_{pk_1k_2} + \alpha_{(k_1k_2)} - c_{pk_3k_2} - \alpha_{(k_3k_2)}]; \tag{13.10}$$

$$\alpha^{r_3}_{pk_3} = [З^{r_2}_{pk_1} - З^{r_3}_{pk_3}] + [c_{pk_1k_2} + \alpha_{(k_1k_2)} - \\ - c_{pk_3k_2} - \alpha_{(k_3k_2)}]. \tag{13.11}$$

Taking these expressions into account we see that the economic nature of the shadow prices of the use of a product of the complementary industries (w_{pk}) and of the efficiency of a production activity (α^r_{pk}) are similar to the economic nature of the corresponding shadow prices of the specialisation industries. But, since the conditions of the siting of separate production activities of the complementary group in areals differ from those of the location of the specialisation industries (for the former it is permissible to build several similar facilities in one areal, but for the latter it is impermissible to build more than one production activity), the prices concerned will have a different structure. It is not enough, in order to interpret it, to indicate the areal where the marginal facility is located; the variant of the production activity must also be indicated. Taking that into account we can give the following description of the shadow prices of the complementary industries being considered.

In the areal in which the marginal variant of a production activity* is located, the shadow price of the use of the product is equal to the size of the region's total unit inputs and the sum total of the unit costs of the feedback arising from development in the region of elements of the economy serving this variant, i.e. expression (13.7). This shadow price forms part of the structure (a) of the shadow prices of the efficiency of the variants of making this product, i.e. expressions (13.9), (13.10), and (13.8), (13.11). This means that the level of the region's total unit costs of producing output by the marginal variant affects the outlays of all consumers of the product and the efficiency of the operation of analogous production activities wherever they are located.

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* The variant that has a zero shadow price of efficiency in the optimum plan. In the situations being considered only those cases are analysed in which capacities of the production activities are below the upper limit.

In an areal in which there is no local production activity, the shadow price of the output utilisation is determined by the level of the region's total unit costs of making this product by the marginal variant and the level of the total costs of transporting a unit of output from the areal where this variant is located.

In an areal in which there is one or more effective variants of a production activity*, the shadow price of the use of the product is determined by the level of the region's total unit costs of making this product by the marginal variant and the difference between the total transport costs of delivering a unit of the product to the marginal areal of consumption from the areal where the marginal variant is located and from the areal concerned, i.e. expressions (13.9) and (13.10).

The shadow price of the efficiency of a variant of the building and siting of a complementary production activity depends on the areals (i) of the location of the effective and marginal variants of output of the product and (ii) of the marginal areal of its utilisation. If the marginal and effective variants of a production activity are located in the same areal, the shadow price is equal to the difference between the region's total unit costs of making the product by these variants, i.e. expression (13.8). If the marginal and effective variants are located in different areals, the shadow price includes the difference between the total transport costs of delivering a unit of the product from one of these areals to the marginal areal of consumption, i.e. expression (13.11).

In order to elucidate the economic nature of the shadow price of the import of a product ($\eta_{p\hat{k}}$), let us consider the following situation.

Situation 3. Suppose that product p is brought into areal of consumption k_2 from region $\hat{k}$ through boundary areal k_4, the limit on imports being fully observed, and a need arising to limit production within the region being considered. Let the conditions of the production of this product be the same as in Situation 2.

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* The variant that has a positive shadow price in the optimum plan. Production capacity under this variant always reaches the upper limit.

Taking expression 4 of the primal solution into account (Diagram 13.1), and also equation 9° of the dual solution (Diagram 13.2), we write:

$$w_{pk_3} = s_{p\hat{k}k_4} + \eta_{p\hat{k}} + c_{pk_4k_2} + \alpha_{(k_4k_2)}. \tag{13.12}$$

Comparing (13.12) with expression (13.9), we find:

$$\eta_{p\hat{k}} = [\mathfrak{Z}^{r_2}_{pk_1} + c_{pk_1k_2} + \alpha_{(k_1k_2)}] - [s_{p\hat{k}k_3} + c_{pk_4k_2} + \alpha_{(k_4k_2)}]. \tag{13.13}$$

By analysing the structure of expression (13.13) we see that the shadow price of the limit on imports of a product that is also made within the region being considered is equal to the difference between the total transport and production inputs of delivering a unit of the product to the marginal areal of consumption from the areal of the location of the marginal variant of its production and from the region of import. It indicates by how much inputs would be reduced in the region as a whole if the limit on importing the product could be raised, i.e. it characterises the efficiency of using the imported product in the region. The value of this shadow price does not here affect consumers' outlays. As in the previous situation the shadow price of the use of a product is determined by the region's total unit costs of making it by the marginal variant and the total transport costs of delivering a unit of it from the areal where this variant is located, as can be seen from substituting expression (13.13) in expression (13.12).

The shadow prices w_{pk} and $\eta_{p\hat{k}}$ will have a different structure if it is either impossible or undesirable to organise local production of the product in the region being considered. This can be demonstrated by analysing the following situation.

Situation 4. Let product p be imported into areal k_3 from regions $\hat{k}_1$ and $\hat{k}_2$. Imports from region $\hat{k}_1$ are made through boundary areal k_1 and from region $\hat{k}_2$ through boundary areal k_2, the limit on imports from region k_1 having been exhausted, but not that for region $\hat{k}_2$.

Then, considering the corresponding expressions of the primal and dual solutions, we can write:

$$w_{pk_1} = s_{p\hat{k}_1 k_1} + \eta_{p\hat{k}_1}, \quad \eta_{p\hat{k}_1} \geqslant 0; \tag{13.14}$$

$$w_{pk_2} = s_{p\hat{k}_2 k_2}, \quad \eta_{p\hat{k}_2} = 0; \tag{13.15}$$

$$w_{pk_3} = s_{p\hat{k}_2 k_2} + c_{pk_2 k_3} + \alpha_{(k_2 k_3)}; \tag{13.16}$$

$$\eta_{p\hat{k}_1} = [s_{p\hat{k}_2 k_2} + c_{pk_2 k_3} + \alpha_{(k_2 k_3)}] - [s_{p\hat{k}_1 k_1} + c_{pk_1 k_3} + \alpha_{(k_1 k_3)}]. \tag{13.17}$$

We see that the shadow price of the use of a product in any areal is determined by the level of the unit costs (of production and transport) of the marginal region of import.* In the marginal areal of consumption, and in the areal where demand is met only by imports from the marginal region, the shadow price is equal to the value of the inputs on delivering a unit of the product from the marginal region of import, i.e. by expressions (13.16) and (13.15). In the areal where demand is met only by imports from the effective region**, the shadow price is equal to the sum of two quantities: namely, (a) the value of the direct inputs on importing of the product from the effective region; and (b) the shadow price of the limit on imports, i.e. expression (13.14). The shadow price of the limit on imports is equal here to the difference between the total unit costs of delivering the product to the marginal areal of consumption from the marginal and effective regions of import, i.e. expression (13.17). This will readily be seen by substituting expression (13.17) in expression (13.14).

The shadow prices of the transport system. These are the shadow prices of the efficiency (a) of the reserve carrying capacity of the section of the system concerned ($\alpha_{(kk')}$) and (b) of the reconstructed section ($\beta_{(kk')}$). In order to understand their economic nature we shall analyse several simple situations of the optimum plan, taking into account the appropriate equations 7 and 8 of the primal solution (Diagram 13.1) and equation 12° of the dual solution (Diagram 13.2).

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* The region with a zero shadow price of the efficiency of import in the optimum plan. In the situations being considered it is assumed that the limit on imports from this region is not fully exhausted.

** The region with a positive shadow price of efficiency of import. The limit on imports from this region is always completely used up.

Situation 5. Let the reserve carrying capacity of section (k_1k_2) be fully utilised, for which freights of product i are carried from areal k_1 to areal k_2 by two routes: namely, by section (k_1k_2) and through areal k_3 by sections (k_1k_3) and (k_3k_2). The reserve capacity of sections (k_1k_3) and (k_3k_2) are not fully exploited in the optimum plan, therefore no need arises to extend them. These constraints can be written as follows:

$$\alpha_{(k_1k_2)} = (c_{ik_1k_3} + c_{ik_3k_2}) - c_{ik_1k_2}. \qquad (13.18)$$

Now assume that the optimum plan provides for reconstruction of section (k_1k_2), with the previous directions of freight traffic. Since $z^r_{(k_1k_2)} = 1$, then $\beta_{(k_1k_2)} \geqslant 0$, and constraint 12° of the dual solution is satisfied as a strict equality. After transforming it, taking (13.18) into account, we find:

$$\beta_{(k_1k_2)} = \Pi^r_{(k_1k_2)}\alpha_{(k_1k_2)} - [S^r_{(k_1k_2)} + \Pi^r_{(k_1k_3)}a^r_{\text{э}(k_1k_2)}w_{\text{э}k_1} + \\ + \Pi^r_{(k_1k_2)}a^r_{\xi(k_1k_2)}w_{\xi k_1}] = \Pi^r_{(k_1k_2)}[c_{ik_1k_3} + c_{ik_3k_2} - c_{ik_1k_2}] - \\ - [S^r_{(k_1k_2)} + \Pi^r_{(k_1k_2)}a^r_{\text{э}(k_1k_2)}w_{\text{э}k_1} + \Pi^r_{(k_1k_2)}a^r_{\xi(k_1k_2)}w_{\xi k_1}]. \qquad (13.19)$$

If we allow for the fact that the reconstructed carrying capacity of the section concerned is not fully exploited, i.e. that $0 < z^r_{(k_1k_2)} < 1$; the following holds:

$$\beta_{(k_1k_2)} = 0;$$

$$\alpha_{(k_1k_2)} = \frac{S^r_{(k_1k_2)}}{\Pi^r_{(k_1k_2)}} + a^r_{\text{э}(k_1k_2)}w_{\text{э}k_1} + a^r_{\xi(k_1k_2)}w_{\xi k_1}. \qquad (13.20)$$

Let us then consider the case in which the limited nature of the reserve carrying capacity of the section affects the location of production activities directly as a limitation on the scale of production in various areals rather than indirectly through the distribution of freights of the product.

Situation 6. Let the reserve carrying capacity of section (k_1k_2) be fully utilised without reconstruction of the section. The demand of areal k_2 for product i is met by imports from areals k_1 and k_3, the reserve carrying capacity of section (k_3k_2) and the productive capacities of activities located in areals k_1 and k_2 not being fully exploited.

In this situation it is necessary to consider equations 1 and 6 of the primal solution (Diagram 13.1) and equation 1° of the dual solution (Diagram 13.2), as well as those employed above. Analysing them as was done in Situation 1, and using the notation employed there, we write:

$$\alpha_{(k_1k_2)} = (\text{З}^{r_3}_{ik_3} - \text{З}^{r_1}_{ik_1}) + (c_{ik_3k_2} - c_{ik_1k_2}). \qquad (13.21)$$

Suppose now that section (k_1k_2) is being reconstructed so that its carrying capacity will be fully exploited, without altering the marginal conditions considered above of the production activities of industry i. We then obtain the shadow price of the variant for reconstructing the section having substituted expression (13.21) in (13.19):

$$\beta_{(k_1k_2)} = \Pi^r_{(k_1k_2)} [\text{З}^{r_3}_{ik_3} - \text{З}^{r_1}_{ik_1} + c_{ik_3k_2} - c_{ik_1k_2}] -$$
$$- [S^r_{(k_1k_2)} + \Pi^r_{(k_1k_2)} a^r_{\text{э}(k_1k_2)} w_{\text{э}k_1} + \Pi^r_{(k_1k_2)} a^r_{\xi(k_1k_2)} w_{\xi k_1}]. \qquad (13.22)$$

Economic analysis of the structure of the expressions considered in Situations 5 and 6 enables us to elucidate the nature of the investigated shadow prices.

The shadow price of the efficiency of the reserve carrying capacity of section of the transport system $(\alpha_{(kk')})$ is formed in each concrete case in accordance with how this reserve affects the distribution of freight traffic and the location of production activities. If the limited nature of the section's reserve carrying capacity makes it necessary to use a more expensive route, then this shadow price is equal to the difference between the transport costs of carrying a unit of the product that limits freight traffic*, i.e. expression (13.18). If the limited character of a section's reserve carrying capacity necessitates its reconstruction, and the new capacity is not fully exploited, then the shadow price is equal to the size of the region's total costs of reconstructing this section, i.e. expression (13.20). If the limited character of a section's reserve carrying capacity directly affects the location of production activities, then the shadow price of this reserve is the sum total of two values: namely, (a) the difference between the region's total costs of making a unit of the product limiting freight traffic by means located in the marginal areals of production; and (b) the difference between the direct transport costs of delivering a unit of the product to the areal of consumption, i.e. expression (13.21). The section of the transport system under consideration here must link one of the marginal areals either directly or through other sections with the areal of consumption.

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* The product that is delivered to the areal of consumption by two or more routes from one or more areals of production, the routes obligatorily including sections whose reserve carrying capacity is fully exploited, including the possibility of reconstructing them.

As will be seen from expressions 4°, 6°, and 7° of the dual solution (Diagram 13.2), shadow price $\alpha_{(kk')}$ affects the size of the shadow prices of all the types of product carried by the section concerned. Consumers' outlays thereby reflect the region's total transport costs of carrying a product, i.e. not only the direct transport costs of carrying specific types of product but also the inputs borne by the region because of the limited character of the reserve carrying capacity of the individual sections. The shadow price of the efficiency of the reconstruction variant for a section also depends on how the section's reserve carrying capacity affects the location of production activities. If the effect is indirect, i.e. through a change in the freight route, then the shadow price is equal to the difference between two quantities: namely, (a) the magnitude of the direct transport costs of carrying the product limiting freight traffic by the expensive route and by the one concerned in the volume of the carrying capacity of the variant of reconstruction; and (b) the size of the region's total costs of carrying out this variant of reconstruction, i.e. expression (13.19). If the effect is direct, through a change of the areals where production activities are located, then the shadow price has a different structure [see expression (13.22)].

Shadow prices of the power system. These are the shadow price of the use of electricity ($w_{эk}$) and the shadow price of the efficiency of the power source ($\beta^{r}_{эk}$). In the model considered a region's power system is represented by large generating stations and transmission lines, assuming that the stations are located together with other elements of the economy by areals, and that the transmission lines connect up these areals. This simplification makes it impossible to bring out a whole variety of intrasectoral conditions of the functioning of the regional power system, and allows us only to trace the system's links with other elements of the region's economy in aggregated form.

The mathematical expression of the constraints on the functioning of the power system in the model concerned is similar to that of the complementary industries. Therefore, if we analyse equations 9 and 10 of the primal solution (Diagram 13.1) and equations 3°, 10°, and 11° of the dual solution (Diagram 13.2), as we did in Situation 2, we can elucidate the structure of the shadow prices of the power system of the model being considered.

The shadow price of the use of electricity in an areal indicates the consumers' expenditure. In the marginal areal this shadow price is established at the level of the region's total unit costs of building the marginal generating station. These costs consist of the total direct inputs, which reflect the specific industrial nature of the generating station and the sum total of the inputs of the feedback that characterises the specific nature of the station's external connections with the region. In the model concerned feedback inputs consist of the cost of developing complementary industries whose output is consumed by the generating stations, and the costs of providing services for the manpower, employed at a given generating station, and the members of the workers' families. In other areals the shadow price is established taking account of the costs of building transmission lines from the marginal generating stations.

The shadow price of the efficiency of separate generating stations is determined by the functioning conditions of the given and marginal stations and those for building transmission lines to the marginal areals of consumption of power.

Shadow prices of manpower. These are the shadow price of the employment of manpower ($w_{\xi k}$), the shadow price of local labour reserves (θ_k), and the shadow price of the region's limit on manpower (θ). How these prices are formed can be found by analysing equations 11 to 13 of the primal solution (Diagram 13.1) and equations 13° and 14° of the dual solution (Diagram 13.2).

Situation 7. Let the manpower limit for the region not be exhausted in the optimum plan. Areals that are not being developed are united into set Q_1; areals in which production activities are being sited, but manpower reserves are not being fully employed go into set Q_2; while areals in which there is a demand for outside labour are put into set Q_3.

In this situation the following holds:

$$\text{for all areals } \theta = 0; \tag{13.23}$$

$$\text{for } k \in Q_1 \ \theta_k = 0, \quad w_{\xi k} \gtrless 0; \tag{13.24}$$

$$\text{for } k \in Q_2 \quad \theta_k = 0,$$

$$w_{\xi k} = \frac{1}{1-\varepsilon_k}\Big(s_k + \sum_i \gamma_{ik} w_{ik} + \sum_p \gamma_{pk} w_{pk} + \\ + \gamma_{\text{э}k} w_{\text{э}k} + \sum_\nu \gamma_{\nu k} w_{\nu k}\Big)\ ; \tag{13.25}$$

$$\text{for } k \in Q_3 \ \theta_k = (\Delta s_k - s_k) + \Big(\sum_\nu \Delta\gamma_{\nu k} w_{\nu k} - \\ - \sum_\nu \gamma_{\nu k} w_{\nu k}\Big); \tag{13.26}$$

$$w_{\xi k} = \frac{1}{1-\varepsilon_k}\Big(\Delta s_k + \sum_i \gamma_{ik} w_{ik} + \sum_p \gamma_{pk} w_{pk} + \\ + \gamma_{\text{э}k} w_{\text{э}k} + \sum_\nu \Delta\gamma_{\nu k} w_{\nu k} + \theta\Big). \tag{13.26a}$$

By analysing the structure of these expressions we see that the shadow price of the employment of manpower in a development areal indicates the size of the region's total inputs on providing services for the population of that areal. It characterises the inputs both on providing services for the population, employed in the production activities under consideration, and on providing for the members of families not employed in these activities. The inputs consist of two amounts: namely, (a) the direct inputs on building elements of the social infrastructure and housing fund (s_k or Δs_k); and (b) the grand total of inputs on the feedback arising in the region in connection with the building of production enterprises whose output is consumed by the population. In the model under consideration this grand total includes the inputs on building the specialisation and complementary industries $\big(\sum_i \gamma_{ik} w_{ik} + \sum_p \gamma_{pk} w_{pk}\big)$, on power supply $(\gamma_{\text{э}k} w_{\text{э}k})$.

and on exploiting local resources ($\gamma_{\nu k}w_{\nu k}$ or $\Delta\gamma_{\nu k}w_{\nu k}$), i.e. expressions (13.25) and (13.26).

In an areal in which only local labour is employed, this shadow price is equal to the value of the region's total costs of providing services for the local population, i.e. expression (13.25). In an areal in which both local and outside labour is employed, the shadow price is equal to the region's total inputs on settling the outside labour, i.e. expression (13.26). There is another shadow price in this areal, as well: namely, that of the pool of local labour θ_k, which is equal to the difference between the region's total costs of providing services (a) for the local population and (b) for that attracted to the areal.

Now let the limit on manpower in the optimum plan for the region as a whole be fully exhausted, without the conditions for the allocation of labour having been altered. According to the formalisation of the problem the variants considered for the output of production are differentiated by indicators of capital and labour intensity, in addition to other characteristics. Given a manpower shortage for the region as a whole there is a product i in the optimum plan that is made by both capital-intensive and labour-intensive variants. Suppose that the productive capacities of these variants are not fully utilised in the optimum plan, and that they are located in the boundary areals k_1 and k_2 through which product i is shipped to region k. For that k_1 and $k_2 \in Q_3$.

By analysing the equations given above for the employment of labour in the primal and dual solutions, and the equations considered in Situation 1, we can write:

$$\theta = \frac{1}{\dfrac{a_{\xi i}^{r2}}{1-\varepsilon_{k_2}} - \dfrac{a_{\xi i}^{r1}}{1-\varepsilon_{k_1}}} \left[\left(\frac{S_{ik_1}^{r1}}{A_{ik_1}^{r1}} - \frac{S_{ik_2}^{r2}}{A_{ik_2}^{r2}} \right) + \right.$$

$$+ \left(\sum_j a_{ji}^{r1} w_{\text{э}k_1} - \sum_j a_{ji}^{r2} w_{\text{э}k_2} \right) + \left(\sum_p a_{pi}^{r1} w_{pk_1} - \right.$$

$$\left. - \sum_p a_{pi}^{r2} w_{pk_2} \right) + \left(a_{\text{э}i}^{r1} w_{\text{э}k_1} - a_{\text{э}i}^{r2} w_{\text{э}k_2} \right) +$$

$$+ \left(\sum_\nu a_{\nu i}^{r1} w_{\nu k_1} - \sum a_{\nu i}^{r2} w_{\nu k_2} \right) +$$

$$+\left(\frac{a_{\xi i}^{r_1}}{1-\varepsilon_{k_1}}\sum_i \gamma_{ik_1}w_{ik_1}-\frac{a_{\xi i}^{r_2}}{1-\varepsilon_{k_2}}\sum_i \gamma_{ik_2}w_{ik_2}\right)+$$

$$+\left(\frac{a_{\xi i}^{r_1}}{1-\varepsilon_{k_1}}\sum_p \gamma_{pk_1}w_{pk_1}-\frac{a_{\xi i}^{r_2}}{1-\varepsilon_{k_2}}\sum_p \gamma_{pk_2}w_{pk_2}\right)+$$

$$+\left(\frac{a_{\xi i}^{r_1}}{1-\varepsilon_{k_1}}\gamma_{\text{э}k_1}w_{\text{э}k_1}-\frac{a_{\xi i}^{r_2}}{1-\varepsilon_{k_2}}\gamma_{\text{э}k_2}w_{\text{э}k_2}\right)+$$

$$+\left(\frac{a_{\xi i}^{r_1}}{1-\varepsilon_{k_1}}\sum_\nu \Delta\gamma_{\nu k_1}w_{\nu k_1}-\frac{a_{\xi i}^{r_2}}{1-\varepsilon_{k_2}}\sum_\nu \Delta\gamma_{\nu k_2}w_{\nu k_2}\right)+$$

$$+\left(\frac{a_{\xi i}^{r_1}}{1-\varepsilon_{k_1}}\Delta s_{k_1}-\frac{a_{\xi j}^{r_2}}{1-\varepsilon_{k_2}}\Delta s_{k_2}\right)\Bigg]; \qquad (13.27)$$

for $k \in Q_2$

$$w_{\xi k}=\frac{1}{1-\varepsilon_k}\Bigg(s_k+\sum_i \gamma_{ik}w_{ik}+\sum_p \gamma_{pk}w_{pk}+$$

$$+\gamma_{\text{э}k}w_{\text{э}k}+\sum_\nu \gamma_{\nu k}w_{\nu k}+\theta\Bigg); \qquad (13.28)$$

for $k \in Q_3$

$$\theta_k=(\Delta s_k-s_k)+\left(\sum_\nu \Delta\gamma_{\nu k}w_{\nu k}-\sum_\nu \gamma_{\nu k}w_{k\nu}\right); \qquad (13.29)$$

$$w_{\xi k}-\frac{1}{1-\varepsilon_k}\Bigg(\Delta s_k+\sum_i \gamma_{ik}w_{ik}+\sum_p \gamma_{pk}w_{pk}+$$

$$+\gamma_{\text{э}k}w_{\text{э}k}+\sum_\nu \Delta\gamma_{\nu k}w_{\nu k}\Bigg). \qquad (13.29a)$$

By comparing these expressions with (13.23) to (13.26) inclusive, we find that the shadow price of the employment of labour in any areal, given a shortage of manpower in the region, will include the shadow price of the region's manpower limit. In that way the conditions for exploiting scarce economic resources in all areals are evened out.

This last shadow price indicates the saving in inputs for the region as a whole of using the labour-intensive variants of production rather than the capital-intensive one. If these variants apply to a specialisation industry and are located in the various boundary areals through which the product is brought into one and the same region, then the shadow price is equal to the grand total of two quantities: namely, the difference between the total unit costs of making the product by these variants; and (b) the difference between the region's total unit costs of providing services for the population in the areals concerned, corrected by a factor indicating the ratio of the indicators of labour-intensity of these variants, i.e. expression (13.27). If the marginal production activities (as regards employment of manpower) are located in interior areals of the region, then the shadow price of the manpower limit will also include the difference between the total transport costs of delivering the product to the marginal consumer from these areals. If, however, the production activities are located in one areal, then the shadow price will only include the difference between the total unit costs of making the product, etc. Note that a labour shortage in the region will be reflected only in the value of the shadow price of employment and will not affect the shadow price of the areals' local labour reserves. This will be readily seen by comparing expressions (13.26) and (13.29).

Shadow prices of the local resources of areals (*land and water*). These are the shadow prices (a) of the exploitation of a resource ($w_{\nu k}$) and (b) of the efficiency of the category of resource ($\alpha^{\mu}_{\nu k}$). The economic nature of these shadow prices can be understood by analysing equations 14 and 15 of the primal solution (Diagram 13.1), and equation 15° of the dual solution (Diagram 13.2).

Situation 8. Let categories μ_1 and μ_2 of resource ν in areal k be exploited in the optimum plan. Then $y^{\mu_1}_{\nu k} = M^{\mu_1}_{\nu k}$, and $y^{\mu_2}_{\nu k} < M^{\mu_2}_{\nu k}$. By analysing these equations we write:

$$w_{\nu k} = 3^{\mu_2}_{\nu k}; \qquad (13.30)$$

$$\alpha^{\mu_1}_{\nu k} = 3^{\mu_2}_{\nu k} - 3^{\mu_1}_{\nu k}. \qquad (13.31)$$

This means that the shadow price of exploiting a local resource in areal k is equal to the quantity of the direct costs of exploiting the marginal category of this resource, and indicates the consumers' outlays on it. The shadow price of the efficiency of the category of the resource is equal to the differences between the direct costs of exploiting the marginal category of this resource and the given one, and indicates the rent obtained by the region from exploiting the best categories of resources, given their rational allocation between all consumers.

We can now review the optimality criterion of the dual solution. The values of the objective functions of the primal and dual solutions are equal, of course, in the optimum plan. The objective function of the primal solution indicates the grand total of the region's adjusted costs of meeting the targets for production and export of output of the specialisation industries, given proportional development of all the elements of the economy and parts of the area concerned. That is to say, the economic nature of the objective function of the dual solution is also one of inputs. By analysing its structure (i.e. expression 16°), we see that the terms corresponding to the target for shipping output of the specialisation industries have a positive sign, while the terms corresponding to the limitation (a) on the operational scale of production activities and of the facilities of the infrastructure in various areals, (b) on the limit of utilisation of the previously built underutilised capacities of the infrastructure, and (c) on the limit on exploitation of local and national resources in separate areals or in the region as a whole, have a negative sign.

Transposing the terms with negative signs from the objective function of the dual solution to that of the primal solution, we write:

$$\sum_{i,\,\bar{k}} B_{i\bar{k}} u_{i\bar{k}} = \Big[\sum_{k,\,i,\,r} S^r_{ik} x^r_{ik} + \sum_{k,\,p,\,r} s^r_{pk} y_{pk} +$$

$$+ \sum_{k,\,r} S^r_{\partial k} x^r_{\partial k} + \sum_{i,\,k,\,k'} c_{ikk'} v_{ikk'} + \sum_{p,\,k,\,k'} c_{pkk'} v_{pkk'} +$$

$$+ \sum_{k,\,k'} s_{\partial kk'} v_{\partial kk'} + \sum_{r,\,(kk')} S^r_{(kk')} z^r_{(kk')} +$$

$$+ \sum_{k} s_k l_k + \sum_{k} \Delta s_k \, \Delta l_k + \sum_{k,\,\mu,\,\nu} s^{\mu}_{\nu k} y^{\mu}_{\nu k} \Big] +$$

$$+ \Big[\Big(\sum_{k,\,p,\,r} N^r_{pk} \alpha^r_{pk} + \sum_{k,\,i} \beta_{ik} + \sum_{(kk')} \Pi_{(kk')} \alpha_{(kk')} +$$

$$+\sum_{(kk')}\beta_{(kk')}+\sum_{r,\,k}\beta^{r}_{\text{э}k}\Big)+\Big(\sum_{p,\,\hat{k}}D_{p\hat{k}}\eta_{p\hat{k}}+$$

$$+\sum_{k}L_k\theta_k+L\theta+\sum_{k,\,\nu,\,\mu}M^{\mu}_{\nu k}\alpha^{\mu}_{\nu k}\Big)\Big]. \qquad (13.32)$$

The structure of this expression can be interpreted as follows. The region's total costs of meeting the targets for production and export of output of the specialisation industries consist of two kinds: namely, (a) the grand total of the direct costs of building and operating the elements of the economy that make it possible to meet these targets (the objective function of the primal solution or the expression in the first square brackets); and (b) the grand total of the receipts from all consumers of the types of product, service, and resource under review (the expression in the second square brackets). These receipts are broken down in turn (a) into the rents received by the region as owner of the resources (local and national used up in development) and of the infrastructure, and (b) into the profit made by the industries through rational siting of their facilities in the region.

2. SHADOW PRICES OF THE SIMPLIFIED MODEL OF A COMPLEX'S SPATIAL STRUCTURE

The mathematical expression of the model being investigated is given in Diagram 13.3. The main objects of investigation in this model are the enterprises of the specialisation industries, elements of the sub-areal's infrastructure (heat and power stations; water installations and purification plant), transmission lines, separate sections of the transport system, local resources of the sub-areals (water and land), and the population of the towns being formed.

Here, as in the full model (see Chapter 6), the solution of the siting of enterprises of the specialisation industries is given: constraint 12 (Diagram 13.3) corresponds to constraint (6.2). In contrast to the full model, however, the input-output matrices of the production and distribution of the output of these industries are not considered, since they are similar to the corresponding constraints of the model treated above, and their shadow prices will have the same structure. In that case the transport costs will be taken into account in the indices of the direct costs of building the sited enterprises, and the simplified model will include constraint 6 (Diagram 13.3), which indicates the existence of a transport link between each pair of enterprises considered.

In contrast to the full model, the condition for the building and operation of the complementary industries is not considered, i.e. equations (6.3), (6.4), (6.9), and (6.10) are omitted. On the other hand, the condition of the building and operation of the complex's infrastructure is given in greater detail: the specific nature of the functioning of the facilities of a sub-areal's infrastructure, the output of which is immobile, is disclosed through constraints 3 and 4; constraints 9 to 11 inclusive show how the complex's power system is formed, while constraints 7 and 8, which correspond to constraints (6.6) to (6.8) inclusive of the full model, represent the separate sections of the transport system.

The constraints on the employment of labour and local resources of the complex are given in simplified form in the model being studied. Instead of constraints (6.5) and (6.11) to (6.13), which bring out the specific character of the forming of the employment input-output matrix for a sub-areal, one constraint only is considered for the forming of the total population of a sub-areal: namely, constraint 5 (Diagram 13.3). This simplification is due to the fact that the effect on the shadow price of the employment of labour of the difference between the use of local and outside labour was already considered when the shadow prices of the model of the spatial structure of the system of complexes were investigated, while the effect of the difference in employment of male and female labour was shown earlier (see Chapter 11). For that reason the simplified model shows the constraints on using a sub-areal's territory, constraints 1 and 2 corresponding to constraints (6.14) to (6.18) inclusive. The sanitation requirements for the siting of enterprises [constraint (6.19)] are omitted in the simplified model. The optimality criterion has the same economic nature.

As before, so as to bring out the economic sense and structure of the different shadow prices, we examine how the conjugate equations of the primal and dual solutions (Diagram 13.4) for the main objects of investigation are satisfied in the optimum plan.

Shadow prices of land. These are the shadow price of the utilisation of land (w_q), and the shadow price of the efficiency of the category of land (v_q^{μ}). In order to elucidate their economic nature we analyse equations 1 and 2 of the primal solution (Diagram 13.3) and equation 2° of the dual solution (Diagram 13.4).

Situation 9. Suppose that one or more enterprises of the specialisation industries are sited in sub-areal q, i.e. that its land is occupied by industrial and civil building, land of μ_1 category being completely utilised and that of μ_2 category partially so, i.e.

$y_q^{\mu_1} = B_q^{\mu_1}$, $0 < y_q^{\mu_2} < B_q^{\mu_2}$. Considering these equations we write:

$$w_q = 3_q^{\mu_2}; \tag{13.33}$$

$$v_q^{\mu_1} = 3_q^{\mu_2} - 3_q^{\mu_1}. \tag{13.34}$$

That is to say, the shadow price of the utilisation of a sub-areal's land is equal to the value of the direct costs of utilising a unit of land of the marginal category.* It indicates the compensation costs of the various facilities built in the sub-areal to make up for the farmland used for industrial and civil building. But the shadow price of the efficiency of the different categories is equal to the difference between the direct costs of utilising a unit of the marginal category of land and a unit of the land under consideration. It also indicates the rent obtained by the complex from the sited facilities, as owner of this type of resource.

Shadow prices of a sub-areal's infrastructure. These are the shadow price of the use of services of the infrastructure ($u_{\tau q}$) and the shadow price of the efficiency of a variant of building the infrastructure ($v_{\tau q}^{r}$). The conditions of the distribution of the product (services) of a sub-areal's infrastructure and that of the building and functioning of the separate facilities are similar to the analogous constraints of the complementary industries in the model of the specialisation of a region's system of complexes previously considered. The difference is that the product of the complementary industries is moveable while the products (services) of the facilities of the sub-areal infrastructure are consumed where they are located. This peculiarity is reflected in both the structure and the sense of the corresponding shadow prices. Analysis of equations 3 and 4 of the primal solution (Diagram 13.3) and equation 3° of the dual solution (Diagram 13.4) helps to clarify this.

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* The land that has a zero shadow price of efficiency in the optimum plan. Here we consider the case when this land is not wholly used for building.

Situation 10. Let one or more enterprises be located in sub-areal q and a need rise for the product (services) of its infrastructure, which is met through the functioning of two variants (facilities) r_1 and r_2, so that $y_{\tau q}^{r} = B_{\tau q}^{r1}$, $0 < y_{\tau q}^{r2} < B_{\tau_q}^{r_2}$. Employing these equations we write:

$$u_{\tau q} = s_{\tau q}^{r_2} + a_{\tau}^{r_2} w_q + \sum_{\tau'} a_{\tau'\tau}^{r_2} u_{\tau' q} +$$

$$+ \sum_{d} \Gamma_{r_2 \tau q}^{d} w_d + \vartheta_{\tau}^{r_2} w_q^{\vartheta} = 3_q^{r_2 *}; \qquad (13.35)$$

$$v_{\tau q}^{r_1} = 3_{\tau q}^{r_2} - 3_{\tau q}^{r_1}, \ v_{\tau q}^{r_2} = 0. \qquad (13.36)$$

Economic analysis of the structure of these expressions indicates that if two variants of one type of a sub-areal's infrastructure are built in one sub-areal, the shadow price of using their product (services) will be determined by the level of the complex's total unit costs of building and operating the marginal variant, and will consist of the direct unit costs of building and operating the marginal variant ($s_{\tau q}^{r2}$) plus the grand total of the unit inputs of the feedback arising through utilisation of the sub-areal's resources or the product (services) of other types of infrastructure. In the model concerned the input of the feedback is (a) on compensating for farmland taken up by industrial projects ($a_{\tau_q}^{r_2} w_q$), (b) on using the product of the other types of the sub-areal's infrastructure $\left(\sum_{\tau'} a_{\tau'\tau}^{r^2} u_{\tau' q}\right)$, (c) on developing the transport system $\left(\sum_{d} \Gamma_{r_2\tau q}^{d} w_d\right)$, and (d) on power supply $(\mathcal{J}_{\tau}^{r_2} w_q^{\vartheta})$. The shadow price of utilisation indicates the outlays of consumers of this type of product (service) of the infrastructure.

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* Here, as before, the symbol 3_q^r is used to condense the writing of the separate expressions.

The shadow price of the efficiency of a variant of building a facility of the sub-areal infrastructure is equal to the difference between the complex's total unit costs of building the marginal and given variants. By comparing these shadow prices with the corresponding shadow prices of the complementary industries we see that they are determined solely by the sub-areal's conditions of the building and operation of these production activities.

The shadow price of providing services for the population (u_q). As already mentioned, the constraints on the employment of labour are not investigated in the model of a complex's spatial structure, but only those of providing the population with services in a sub-areal. The assumption made of the proportionality of the population of the towns being formed to the size of the labour force employed in the enterprises of the specialisation industries allows us to represent the total costs of providing for the population in the costs of siting the enterprises. The scale of the model is thus reduced by excluding constraint 5 (Diagram 13.3), but it then becomes more difficult to elucidate the size and structure of the total costs of providing services for the population of a sub-areal as a whole. The population of the towns being formed is, therefore, an autonomous object of investigation in the model being considered. In order to bring out the structure and sense of this shadow price we allow for the fact that equation 5 of the primal solution (Diagram 13.3) and equation 4° of the dual solution (Diagram 13.4) are met in the optimum plan.

Situation 11. Let one or more enterprises be located in sub-areal q and a need rise to build a town. For this sub-areal we write the following as the basis for analysing these equations:

$$u_q = s_q + \gamma w_q + \sum_{\tau} \gamma_\tau u_{\tau q} + \mathcal{J} w_q^{\vartheta}. \qquad (13.37)$$

It will be clear from this expression that the shadow price of providing the sub-areal's population with services indicates the magnitude of the complex's total costs of building a town. These inputs comprise (a) the direct inputs, i.e. the volume of capital investment on building the aggregate of facilities of the social infrastructure, including housing (s_q), and (b) the grand total of the inputs of the feedback. The latter consist of the inputs to compensate for farmland used for civil construction (γw_q), on building facilities of the sub-areal infrastructure serving the population $\left(\sum_{\tau} \gamma_\tau u_{\tau q}\right)$, and on power supply, i.e. on building transmission lines ($\text{Э} w^{\text{э}}_q$).

Shadow prices of the complex's power system. These are the shadow price of using electricity ($w^{\text{э}}_q$), the shadow price of distributing the electricity generated by power stations ($w^{\text{э}}_g$), and the shadow price of the efficiency of a generating station ($v^{\text{э}}_g$). The constraints on the functioning of the power system are represented in the model being investigated by the input-output matrices of the distribution of electricity from the individual sources, the distribution of electricity in sub-areals and the limitations on the use of reserve generating capacities in previously built power plants. In contrast to the model of the spatial structure of a region's system of complexes, the constraints on the building and operation of new generating stations are not taken into account here, and their sites do not coincide with the location (by sub areals) of the consumers. The shadow prices of the power system will, therefore, have a different structure in the given model. We examine them by analysing equations 9, 10, and 11 of the primal solution (Diagram 13.3) and expressions 5° to 8° inclusive of the dual solution (Diagram 13.4).

Situation 12. Let sub-areal q be supplied with power from two sources g_1 and g_2. The reserve capacity of these generating stations is fully utilised, there are no possibilities of further development of source g_1, and it is necessary to extend the capacity of source g_2. For that situation, the following holds:

$$w^{\text{э}}_q = s_{g_2 q} + s_{g_2}; \tag{13.38}$$

$$w^{\text{э}}_{g_1} = s_{g_2} + s_{g_2 q} - s_{g_1 q}, \quad v^{\text{э}}_{g_1} = s_{g_2} - c_{g_1} + s_{g_2 q} - s_{g_1 q}; \tag{13.39}$$

$$w^{\text{э}}_{g_2} = s_{g_2}, \quad v^{\text{э}}_{g_2} = s_{g_2} - c_{g_2}. \tag{13.40}$$

As will be seen, the shadow price of the utilisation of electricity in a sub-areal is established at the level of the complex's total unit costs of building the transmission lines to carry current from the marginal generating station*, i.e. expression (13.38).

The shadow price of the distribution of current from the marginal source is equal to the amount of the direct costs of generating power by new capacities, and of the effective generating source, i.e. by the difference between two quantities: namely, (a) the complex's total unit input on generating and transmitting electricity from the marginal source to the limiting consumer (sub-areal); and (b) the total unit costs of transmitting power to this consumer from the given generating station, i.e. expression (13.39).

The shadow price of the efficiency of previously built capacity of the marginal generating station is equal to the difference between the costs of generating electricity by new and existing capacities [expression (13.40)] and the shadow price of the efficiency of the existing capacities of the effective generating source** is equal to the difference between the complex's total unit costs of generating and transmitting electricity to the marginal consumer (sub-areal) from the given power source and the effective one.

Shadow prices of the transport system. These are the shadow price of exploitation of a section's carrying capacity (w_d), the shadow price of the efficiency of its reserve carrying capacity (v_d), and the shadow price of the transport links of enterprises ($w_{rjq}^{r'j'q'}$). These shadow prices have a different sense and structure than the analogous ones in the simplified model of the spatial structure of a region's system of complexes, since the transport system is represented differently in this model, in which only those sections of the transport system are examined whose reserve carrying capacity is insufficient to carry the intracomplex freights. These sections are found before solution of the problem by analysing the possible transport links of the enterprises being sited. To elucidate the economic nature of these shadow prices we analyse equations 6, 7, and 8 of the primal solution (Diagram 13.3) and equations 9° and 10° of the dual solution (Diagram 13.4).

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* The power station at which the previously built reserve capacity is fully utilised and there is a need to build new capacity.

** The power station at which the existing reserve capacity is fully utilised and there is no possibility of building new capacity.

Situation 13. Let the reserve carrying capacity of section d of the transport system be fully utilised in the optimum plan, and let it be necessary to reconstruct this section. After examining the expressions above, we write:

$$v_d = s_d, \; w_d = v_d. \tag{13.41}$$

This means that if a section's reserve capacity is fully utilised in the optimum plan and there is a need to reconstruct it, the shadow price of exploiting its capacity will be established at the level of the inputs needed to reconstruct it. In order to elucidate the economic sense of the shadow price of transport links ($w_{rjq}^{r'j'q'}$), it is necessary first to analyse the shadow prices of enterprises.

The shadow price of enterprise (u_j) corresponds to the assignment for siting it within the complex (constraint 12 of Diagram 13.3). This constraint is not introduced into the model for all enterprises. It depends on constraint 6 (Diagram 13.3), which indicates the existence of intracomplex transport links between the pairs of enterprises being considered. The whole aggregate of enterprises located in the complex is divided into two groups according to this characteristic: namely, (a) enterprises without intracomplex links for delivering output; and (b) enterprises with such links. Constraint 6 (Diagram 13.3) is absent for all enterprises of the first group, while constraint 12 (Diagram 13.3) is absent for some of the enterprises of the second group, which may be either suppliers or consumers of goods. The assignment for siting these enterprises in the complex is put into the model indirectly—through the constraint on the connection with suppliers (consumers), i.e. constraint 6 (Diagram 13.3). The structure and sense of the shadow prices u_j and $u_{rjq}^{r'j'q'}$ will depend on which group the enterprises being considered belong to. To establish that we analyse equations 6 and 13 of the primal solution (Diagram 13.3) and equation 1° of the dual solution (Diagram 13.4).

Situation 14. Let enterprise j, belonging to the first group, be located in sub-areal q, i.e. $x_{jq} = 1$. In this situation the following holds:

$$u_j = S_{jq} + A_j w_q + \sum_{\tau} A_{\tau j} u_{\tau q} +$$
$$+ \alpha L_j u_q + \sum_{d} \Gamma^{d}_{rjq} w_d + \text{Э}_j w^{\text{э}}_q. \tag{13.42}$$

From an examination of the equation's structure we see that an enterprise's shadow price indicates the amount of the total costs of building and operating it within the complex. It consists of two types of input: namely, (a) direct costs of building the enterprise (S_{jq}); and (b) the grand total of the costs of the feedback arising in the complex because of the need to develop elements of its economy linked with this enterprise. In the model being examined the second inputs are: the costs of compensating for farmland taken to build the given enterprise ($A_j w_q$); and the costs of building facilities of the sub-areal infrastructure ($A_{\tau j} u_{\tau q}$), providing for the population ($\alpha L_j u_q$), developing the transport system ($\sum_d \Gamma^{d}_{rjq} w_d$), and on power supply ($\text{Э}_j w^{\text{э}}_q$).

Situation 15. We now consider enterprises j_1 and j_2 belonging to the second group, enterprise j_1 being located in sub-areal q_1 and enterprise j_2 in sub-areal q_2. Enterprise j_1 delivers goods to enterprise j_2 by variant r_1 of the transport system. The problem of siting enterprise j_1 is thus introduced into the model both explicitly and indirectly (through constraint 6) in the siting of enterprise j_2. The following holds for this situation:

$$u_{j_1} = S^{r_1}_{j_1 q_1} + A_{j_1} w_{q_1} + \sum_{\tau} A_{\tau j_1} u_{\tau q_1} +$$
$$+ \alpha L_{j_1} u_{q_1} + \sum_{d} \Gamma^{d}_{r_1 j_1 q_1} w_d + \text{Э}_{j_1} u^{\text{э}}_{q_1} + w^{r_1 j_2 q_2}_{r_1 j_1 q_1}; \tag{13.43}$$

$$w^{r_1 j_2 q_2}_{r_1 j_1 q_1} = S^{r_1}_{j_2 q_2} + A_{j_2} w_{q_2} + \sum_{\tau} A_{\tau j_2} u_{\tau q_2} + \alpha L_{j_2} u_{q_2} +$$
$$+ \sum_{d} \Gamma^{d}_{r_1 j_2 q_2} w_d + \text{Э}_{j_2} w^{\text{э}}_{q_2}. \tag{13.44}$$

Analysis of these expressions shows that the shadow price of the transport links in these conditions performs the role of the shadow price of the consumer-enterprise [expression (13.13)] while the shadow price of the supplier-enterprise indicates the sum of the total costs of siting the two enterprises, i.e. the supplier and the consumer-enterprise.

Finally, we examine the optimality criterion of the dual solution. As before we compare the values of the objective functions of the primal and dual solutions in the optimum plan. Transposing the negative terms from the function of the dual solution to that of the primal solution, we write:

$$\sum_{j} u_j = \Big(\sum_{q,\, j,\, r} S^r_{jq} x^r_{jq} + \sum_{q,\, \mu} 3^{\mu}_q y^{\mu}_q + \\ + \sum_{q} s_q h_q + \sum_{q',\, q} s_{q'q} y_{q'q} + \sum_{g,\, q} s_{gq} y_{gq} + \\ + \sum_{g} c_g y_g + \sum_{g} s_g \, \Delta y_g + \sum_{d} s_d \, \Delta z_d \Big) + \\ + \Big(\sum_{q,\, \mu} B^{\mu}_q v^{\mu}_q + \sum_{q,\, r,\, \tau} B^r_{\tau q} v^r_{\tau q} + \sum_{d} B_d v_d + \sum_{g} B_g v^{\partial}_g \Big). \qquad (13.45)$$

From examination of the structure of expression (13.45) we see that the grand total of the direct costs of siting and building the given set of enterprises of the specialisation industries within the complex consists of the following: the direct sectoral costs of building the enterprises plus the complex's costs of building the infrastructure and exploiting local resources (the expression in the first set of round brackets is the objective function of the primal solution); and the grand total of the rents received by the complex from rational allocation of local resources and utilisation of the infrastructure's capacities (the expression in the second set of round brackets).

* * *

Analysis of the shadow prices of these simplified spatial models of a complex thus demonstrates that the economic sense and structure of each shadow price is determined by the concrete constraints of the models being studied. In spite of the differences, however, the shadow prices of the analogous constraints of the various models and those of the analogous constraints of one model have features in common that make it possible to group them, as follows:

(1) the shadow prices of the assignments (a) for shipping out products of the specialisation industries $[(u_{i\bar{j}})$ in the

first model] and (b) for siting enterprises of the specialisation industries [(u_j) in the second model];

(2) the shadow prices of the use of output (resources, services): in the first model w_{ik}, w_{pk}, $w_{\text{э}k}$, $w_{\xi k}$, $w_{\nu k}$; in the second model w_q, $u_{\tau q}$, u_q, w_d, $w_q^{\text{э}}$, $w_g^{\text{э}}$;

(3) the shadow prices of the efficiency of variants of the location and operation of production activities: in the first model β_{ik}, α_{pk}^r, $\beta_{(kk')}$, $\beta_{\text{э}k}^r$; in the second model $v_{\tau q}^r$; the shadow prices of the efficiency of employing underloaded capacities of existing facilities of the infrastructure: in the first model $\alpha_{(kk')}$ and in the second model v_d, $v_g^{\text{э}}$; the shadow prices of the efficiency of exploiting the best limited national and local resources: in the first model $\eta_{p\hat{k}}$, θ_k, θ; in the second model v_q^{μ}.

The shadow prices of these groups are employed differently in mathematical economic analysis of the results of the solution, and in forming the indices of the information links of the models of our system for long-term territorial industrial planning.

14

ACCOUNTING FOR NON-LINEAR DEPENDENCES IN TERRITORIAL MODELS

O. P. Burmatova and V. Yu. Malov

When various territorial systems are being studied by means of mathematical economic models non-linear conditions of the extremal problems are often met. Their solving by the methods of linear programming inevitably, as a rule, entails eliminating the non-linear dependences of the various indices, which often leads to considerable over-simplifications, with the result that there may be essential distortions of the real behaviour of the system concerned.

Take, for example, the formulation of the problem of optimising the spatial structure of a territorial industrial complex. For this the following are taken as the objects of investigation: enterprises of the specialisation and complementary industries; the area suitable for industrial and civil building; the sections of the transport system; manpower; and the power system. The mathematical economic model of the problem is considered a linear one.

On the whole a linear relation can be assumed between the different indices for a certain type of relationship (e.g. for that between the volume of production and the input of raw materials). But in problems of optimising the organisation of a complex we also come up against very diverse non-linear relations. Thus unit current and capital costs are not identical for various volumes of production. An increase in scale usually makes it possible to reduce current inputs. All forms of current costs can be divided into two groups: namely, (a) constant, and (b) variable relative to the volume of production. Then

$$\varphi(y) = ay + b, \tag{14.1}$$

in which $\varphi(y)$ is the current costs for the whole volume of production;

y is the volume of production;

b is the constant costs relative to the volume of production;

a is the variable costs per unit of output (input of raw materials, basic supplies, etc.)

Dividing equation (14.1) by y and denoting $\frac{\varphi(y)}{y}$ by z, we obtain:

$$\frac{\varphi(y)}{y}=a+\frac{b}{y} \quad \text{or} \quad z=a+\frac{b}{y}. \tag{14.2}$$

Unit capital inputs have a certain dependence on the degree of concentration of production in separate enterprises; and the functions that express the relationship between the grand total of the adjusted costs of the whole volume of output and the volume of production will be characterised by alternating convex and concave segments.

With different values of the level of output, production inputs (both unit and aggregate) will thus vary in non-linear fashion and this relation's inclusion in territorial models will make the objective function non-linear.

Unit inputs of many types of resource are also not independent of the volume of production. Certain specific resources (water, power, labour, land, and so on) of multipurpose significance are treated separately in territorial models as autonomous conditions. It is not always correct to treat their inputs as constant for different scales of production. As production becomes more concentrated, for instance, inputs of land, labour, and certain other resources per unit of output fall. This also necessitates introducing non-linear dependences in that part of the problem's system of constraints that describes the allocation of territorial resources.

One of the objects of investigation in models of a complex, furthermore, is the transport system of the area concerned. There is non-linearity here if the costs of transporting a unit of output depend on the volume of freight.

In territorial problems, moreover, there may be non-linearity linked with the character of the relations between the volume of output and the amount of building work involved in creating the respective production activities, and with the inputs per unit of output depending on the level of development of the forms of social organisation of production (specialisation, joint production, co-operation, and so on).

Consider certain possible approaches to take into account some non-linear dependences in the models of a complex: namely, the relation between production inputs at individual enterprises and the degree of concentration of production in them; the dependence of costs of the production of output on the location of enterprises of different production activities within one and the same sub-areal (the effect of agglomeration of production); and the dependence of the costs of building separate elements of the complex's infrastructure on the volume of production of its enterprises.

The non-linearity of the first two types of dependence can be taken into account by reducing the problem of optimising the formation of a complex to an enumerable one. Then the non-linear character of the relation between inputs and volume of production is taken into account by representing the possibility of developing the enterprises considered as a finite number of variants for each of which there are corresponding values of the objective function, given in tabular form. A lowering of unit inputs with increase in an enterprise's capacity can thus be allowed for.

The following approach can be employed to account for the dependence of inputs for the production of output on the mutual location of the enterprises of various production activities in one and the same industrial sub-areal. This circumstance, of course, can have a definite effect, owing to joint use of the elements of the production, social, and amenity infrastructure. But as new facilities are sited this effect is reduced, so that the saving in inputs from siting a fourth enterprise, for example, in a sub-areal with three already existing enterprises is greater than that from siting a fifth, and smaller than from siting a third enterprise (with two existing ones), and so on. The saving in inputs from the joint siting of a group of enterprises in one sub-areal varies in non-linear fashion. This can be taken into account as follows.

The matrix of the size of the saving in adjusted costs from joint location is given from estimates. Consider the case of a matrix containing information on the size of the inputs for a pair of enterprises only (the method for computing the amount of the saving when more than two enterprises are sited in the same sub-areal is given below). The matrix is given in Diagram 14.1.

In the diagram l_{ij} $(i \neq j;\ i, j = \overline{1, n})$ is the amount of the saving in inputs from joint location of two enterprises

Diagram 14.1.

Matrix of the Savings in Total Inputs from Joint Location of Enterprises in the Same Area

No. of enterprises of different production activities	Number of enterprises ($i \neq j$)							
	1	2	3	...	j	...	$n-1$	n
1	0	l_{12}	l_{13}	...	l_{1j}	...	l_{1n-1}	l_{1n}
2	l_{21}	0	l_{23}	...	l_{2j}	...	l_{2n-1}	l_{2n}
3	l_{31}	l_{32}	0	...	l_{3j}	...	l_{3n-1}	l_{3n}
...	...	...	...	...	...	...	...	...
i	l_{i1}	l_{i2}	l_{i3}	...	l_{ij}	...	l_{in-1}	l_{in}
...	...	...	...	...	...	...	...	...
$n-1$	$l_{n-1,1}$	$l_{n-1,2}$	$l_{n-1,3}$	...	$l_{n-1,j}$	...	0	$l_{n-1,n}$
n	l_{n1}	l_{n2}	l_{n3}	...	l_{nj}	...	$l_{n,n-1}$	0

of different production activities (i and j)* in one sub-areal. The matrix is symmetrical: $l_{ij} = l_{ji}$ (given $i \neq j$). The elements of its main diagonal are equal to zero: $l_{ij} = 0$ with $i = j$ (i.e. when two enterprises belonging to one industry are located in the same sub-areal they are counted as one). It will be evident that some of the values of l_{ij} (given $i \neq j$) may also be zero.

Now suppose that n enterprises are simultaneously located in one sub-areal. To calculate the total saving of inputs from their agglomeration in one area we first arrange the values of l_{ij} (given $i \neq j$; $i, j = \overline{1, n}$) in descending order:

---

* The matrix contains data on the savings in input for all possible pairs of these enterprises. The number of such pairs will be equal to the number of combinations of two from n, i.e. c_n^2.

$$l_{(ij)_1} > l_{(ij)_2} > \ldots > l_{(ij)_{C_n^2}}.$$

We then calculate the following:

$$\bar{l}_1 = \alpha_1 l_{(ij)_1} = \alpha_1 \max_{k=\overline{1, C_n^2}} l_{(ij)_k};$$

$$\bar{l}_2 = \alpha_2 l_{(ij)_2};$$

$$\ldots\ldots$$

$$\bar{l}_k = \alpha_k l_{(ij)_k};$$

$$\ldots\ldots$$

$$\bar{l}_{C_n^2} = \alpha_{C_n^2} l_{(ij)_{C_n^2}},$$

in which α_k ($k = \overline{1, C_n^2}$) is a coefficient characterising how far growth of the saving has been slowed down with increase in the number of enterprises sited in the same area ($\alpha_k \leqslant 1$).

In our example it is taken, for convenience's sake, that

$$\alpha_1 = 1;\ \alpha_2 = \frac{1}{2};\ \alpha_3 = \frac{1}{3};\ \ldots,\ \alpha_k = \frac{1}{k};$$

$$\ldots,\ \alpha_{C_n^2} = \frac{1}{C_n^2}.$$

The summed effect of joint location is

$$E = \sum_{k=1}^{C_n^2} \bar{l}_k = \sum_{k=1}^{C_n^2} \alpha_k l_{(ij)_k} = l_{(ij)_1} +$$

$$+ \frac{1}{2} l_{(ij)_2} + \ldots + \frac{1}{k} l_{(ij)_k} + \ldots + \frac{1}{C_n^2} l_{(ij)_{C_n^2}}.$$

It must be remembered here that with several enterprises of different production activities sited within one areal, two situations are possible, situations that have to be taken into account when the values of l_{ij} are determined: namely,

(1) several enterprises are located in a sub-areal whose territory has not yet been developed;

(2) enterprises have already been built in the sub-areal and new ones are being added.

In both cases the value of l_{ij} (for the same sets i and j) will obviously be different. In the first the siting of enterprises in an undeveloped sub-areal will inevitably entail a raising of the initial costs of building the needed elements of the production and social infrastructure. In the second case, the sub-areal has already been developed to some extent, and individual enterprises are functioning in it; the siting of new enterprises will entail lower inputs because they can use already existing elements of. the infrastructure. The lowering of inputs through the joint location of several enterprises in the same sub-areal will therefore be greater in this case than in the first one, i.e. the saving in inputs l_{ij} will be differentiated by sub-areals according to their degree of development. Since pairs of enterprises (i, j) were taken as the initial combinations,

$$l_{(ij)}=\begin{cases} l^0_{ij}\text{— with the siting of enterprises } i \text{ and } j \text{ in an undeveloped sub-areal;} \\ l'_{ij}\text{— with the siting of enterprises } i \text{ and } j \text{ in a developed sub-areal, with } l'_{ij} > l^0_{ij}. \end{cases}$$

The non-linearity of the function of dependence of inputs on concentrating production activities is allowed for by employing a point approximation of it.

Another approach proposed for accounting for non-linear dependences is based on the possibility of representing the coefficient of the objective function c in the form of the function $c = f(x)$. In the modified model of a complex's spatial organisation this function is written in general form as follows:

$$(c, x) \rightarrow \min;$$

$$c = f(x), \text{ i.e. } c_j = f_j(x_1 \ldots x_n) \, \forall_j,$$

where c_j is the coefficient of the objective function; and
x_j is the rate of functioning of the methods.

The effect of enlarging enterprise j' equals $c_{j'} = f_{j'}(x_{j'})$, when $\frac{df_{j'}}{dx_{j'}} < 0$. This indicates that the inputs to produce a unit of output are lowered as the scale of production rises, i.e. that $c_{j'}$, is a decreasing function of $x_{j'}$.

The effect of territorial concentration of production is represented as a function of two variables:

$$\Delta c_{ij} = f_{ij}(x_i, x_j),$$

where Δc_{ij} is the lowering of inputs associated with siting the enterprises in one sub-areal;
x_i (x_j) is the rate of operation of enterprise i (j) in this sub-areal.

The function Δc_{ij} is an increasing one for both x_i and x_j. Its coefficients are thus represented as

$$c_i' = c_i^0 - \alpha\,\Delta c_{ij};$$

$$c_j' = c_j^0 - (1-\alpha)\,\Delta c_{ij},$$

where c_i^0 (c_j^0) are the inputs for enterprises i (j) when sited separately;
α — is the share of enterprise i in the saving obtained by territorial concentration of production activities i and j.

When the effect from increasing the output of enterprise i' and that of bringing it closer territorially to enterprise j are both realised, the coefficient of the function for enterprise i' is calculated according to the following rule:

$$\bar{c}_{i'} = c_{i'}^0 - \alpha\,\Delta c_{i'j},$$

where

$$c_{i'}^0 = f_{i'}(x_{i'}); \text{ i.e. } \bar{c}_{i'} = f_{i'}(x_{i'}) = \alpha f_{i'j}(x_{i'}, x_j).$$

The potentialities of linear models allow us to take account only of certain forms of change in outputs depending on the rate of operation: namely, when unit inputs rise with an increase in the rate of exploitation of the method.

Such dependences are inherent in the indices characterising the exploitation of land and water sources during the forming of a complex. In that case the model includes all possible variants of the exploitation of resources with the corresponding costs of these variants; and the problem solved is one of linear programming. But the character of the dependence of unit costs of the rate of operation of the method is an opposite one for many elements of the economy. The variants for the building of sections of railway, transmission lines, etc., are of such a character. In this case that part of a variant (with a low rate of operation) with a higher capacity but, of course, lower unit inputs is selected, given linear formulation of the problem. It is not feasible to account for this kind of dependence in a linear model by introducing different variants. With a whole-number (enumerable) formulation these problems are resolved by introducing Boolean variables, but there are still no working programs for solving such large-scale problems, which forces us in practice (in order to solve them in a continuous formulation) to look for other ways of reflecting dependences. One of these ways is to represent the coefficients of the objective function as $c = f(x)$, for which we approximate them to a continuous function.

The practical computations made by using the proposed modification of the notations of the constraints for forming the coefficient of the objective function, provide grounds for supposing it is possible to construct an iterative process for determining the range of possible changes of the plan, and of changes in the objective function's coefficients for problems with a certain type of function $f(x)$.

Both the first and the second ways of accounting for non-linear relations, however, as yet only enable us to solve a comparatively narrow range of problems (with low dimensionality, a small number of varying coefficients of the objective function, etc.), associated with the discrete character of the changes inherent in most of the processes being modelled. The methods proposed here for the purpose must therefore be regarded as possible directions for looking for an adequate representation of the real objects in models of territorial industrial complexes.

15

THE USE OF PATTERN-RECOGNITION TECHNIQUES TO DETERMINE THE BOUNDARIES OF TERRITORIAL INDUSTRIAL COMBINATIONS

I. V. Mymrin and A. M. Shustorovich

When optimising mathematical economic problems of long-term territorial planning are being formulated, it is necessary, as a preliminary, to elucidate the spatial characteristics of the object of investigation. Various taxons are employed, depending on the purpose of the work, but very often they are not adequately substantiated as regards economic geography, because they do not have spatially defined parameters.

For the problem of specialising a region's system of complexes we tried to identify the sets of areals within the territory being studied, with each areal characterised by a certain community of conditions (properties) of economic development.

An areal is taken as internally homogeneous as regards a set of attributes rather than one attribute alone, the combination being characterised by similar integral conditions of the location and operation of the main production activities, and of settlement. These territorial industrial formations vary in area. It is assumed that one or more industrial centres can be formed (or already operates) within each separate areal, and that a combination of adjacent areals (at least two) will form a large territorial industrial complex within a region.

A schematic map of the areals of Siberia had already been compiled which generalised the published data quantifying regional differences as regards regions' economic development. The boundaries of the zones of action of the various adjustment coefficients reflecting spatial variations of the economic indices of the development of new areas according to different conditions and factors (climatic, seismic, transport, and many others) were shown on the map by isolines. The cartographic method, however, did not yield the dessired result; the isolines intersected the studied area of Siberia in various directions, and it was impossible to isolate the complex characteristics of its separate sectors.

Other techniques also used in physico-geographic and economic zoning also leave much room for subjective judgement, and the danger of selecting an erroneous network of conjugated areas is great.

The sector on the forming of territorial industrial complexes of the Institute of Economics and Industrial Organisation tried to distinguish areals by means of the mathematical technique of pattern recognition.

As we know, certain observed phenomena (and objects) are grouped by means of a complicated system of trial-and-error statistical methods (including operations of generalisation, limitation, analysis, synthesis, association with past experience, and so on). The logic of constructing algorithms consists in automatic selection of one of the possible variants of the combination of given objects of recognition on the basis of analysis and synthesis of the attributes characterising them that do not contain direct indications of which possible solution is best. We considered the opportunities of employing this method in situations in which solution of the problem of perceiving geographical phenomena (recognition) called for intuition and great experience on the part of the researchers, i.e. of specialists qualified in the field of economic zoning.

To solve this mathematical problem we took the central districts of Irkutsk Region. The boundaries of this vast territory, with an area of 512,000 square kilometres, coincided in the west, north-west, and south with the Region's administrative boundaries, in the south-east and east with the shores of Lake Baikal, and in the north and north-east with the boundaries of several of the Region's northern districts. It was necessary to divide this area into an invariant system of areals, the shape and size of which were not known in advance. The main features of the formulating of the problem consisted in the following.

(1) A mathematically rigorous description of the desired territorial combination or areal (in our case, a pat-

tern) could not be employed because the information available did not allow the initial taxon to be delineated, and necessitated the classifying of previously unknown patterns. It was necessary, consequently, to employ recognition algorithms based on cluster analysis. As regards the territory of each areal (Ar), however, it was assumed that there were upper and lower limits that could be conventionally designated as $IC \leqslant Ar \leqslant TIC$ (where IC was an industrial centre, and TIC a regional territorial industrial complex).

(2) The area of the territory being studied was immense. Although its given shape and area indicated the existence of a finite system of desired areals, their total number was not known in advance, and might be quite large.

This specific feature of the problem of delineating areals defined the initial limitations, as follows:

(i) the location and number of single objects of recognition were given in advance;
(ii) areals (patterns) did not overlap in space;
(iii) the initial description of the objects of recognition approximated to a finite number of attributes, on which basis the automatic search for the sought-for patterns (areals) was built up.

Sixty settlements in Irkutsk Region were taken as the objects of recognition that could form areals. They were selected so as, on the one hand, to cover the area of the Region being most intensively developed economically and, on the other hand, to reflect the outlook for their future development. A settlement was interpreted as a point in the space of attributes (indicators) reflecting the regional conditions and factors of the territorial organisation of social production, taking into account the prospects for socio-economic development of the area concerned.

Finally, each settlement had its own special qualitative and quantitative characteristics. At the same time, it possessed a system of attributes with a certain level of quantitative values expressing the character of its connections with the surrounding territory, i.e. with its natural, economic, social, and other conditions of economic development. This objective contradiction found expression in an unevenness in the distribution of urban and rural communities; on the other hand, it made it possible to fix their boundaries in time and space.

Not all the information about settlements was formalised, and the preparation of the initial data for solving the recognition problem called for rigorous logical analysis of their suitability for the traditional solution of such problems. In principle, pattern recognition does not limit the total number of objects of recognition, or of the attributes characterising the patterns. It is advisable, moreover, for the number of objects being classified to be quite large, because the system of patterns obtained in the solution will otherwise be extremely sensitive to the choice of attributes and the statistical accuracy of the coefficients of similarity will be low.

In the preliminary stage of formulating the experimental problem 40 indicators were examined. Not all of the known ones characterising settlements could, however, be employed as initial attributes for machine computation. Some of them had identical terms in the conditions; others were complicated constructs, being in fact synthetic indices, and called for data in the computations that were not then available in the regional statistics. Allowing also for the technical possibilities of the programs available for computers, a limited set of attributes of the objects of recognition was determined. These were indices: (a) of location; of soil and climatic conditions (length of the frost-free and vegetation periods; amount of precipitation); (b) of the density of the river system; (c) of the engineering geology conditions (relief, soils, seismicity, permafrost); (d) of the availability of minerals (coal, underground waters, iron ore, non-ferrous metals, chemical raw materials, oil and natural gas); (e) of timber resources (forest cover); (f) of the accessibility of transport (road and rail); and navigable river routes; (g) of the density of population and the size of settlement points; and (h) of the specialisation of hunting and farming.

The set of indices was chosen so as to correspond in principle to the purpose of delineating areals of Irkutsk Region such that the most accessible regional information could be used to solve the problem. The indices reflected the specific nature of a definite area, and called for the same precision and alteration as for the formulation of similar problems for other regions of the USSR.

In our view there are now ample grounds for considering it feasible to draw up an algorithm of the process of defining an informative system of economic zoning indices. The set of attributes mentioned above cannot, therefore, be considered final; it met the purposes of our experimental study, however, and no less important, made it possible to employ the numerical data of geographical maps of this area at this stage of the solution. This methodological feature of our approach needs to be stressed. A map is the language, the initial standpoint, of zoning, and at the same time its end result, in the form of a geometric network of the partition of the area being studied.

For the first experimental solution of the problem we used an algorithm and program compiled in the Power Institute of the Siberian Department of the USSR Academy of Sciences. This algorithm was based on the concept of proximity as computed by Euclidean geometry. The program was constructed so that the two closest objects were put into group E_1 in the first step of the computations. After that the distances of each of the other objects from group E_1 as delineated were recomputed (using equal average arithmetical distances to the constituents of group E_1), and group E_1 was subsequently treated as a single object.

The next steps were carried out similarly, successively uniting objects and groups into new, larger groupings. The criterion adopted to evaluate each grouping of objects was the value of $d = d_1 + d_2$, in which d_1 and d_2 (for group k) were respectively equal to

$$d_1^{(k)} = \sum_{r,\, r'} d\,(E_r,\, E_{r'});$$

$$d_2^{(k)} = \sum_{k} \sum_{x_m,\, x_{m'} \in E_r} d\,(x_m,\, x_{m'}).$$

These formulas characterise the degree of proximity—the first between groups, and the second between the objects within a group. The grouping of objects was completed when R groups corresponding to $min\ d$ had been selected.

The experiment made it possible to delineate 28 groups of settlements (objects) differing in a wide range of economic and geographic characteristics. The areals obtained differed in the number of settlements involved, which indicated the difference in their shapes and size, and also had a differentiated *quantification* of the local conditions and factors of their economic development, allowing for their future prospects.

The first results of the formulation and solution of the problem by the mathematical technique of pattern recognition permitted us to conclude that the method suggested worked quite flexibly, bringing out the spatial differences in the organisation of a specific areal's industry and settlement, all of which provided the basis for a study of certain methodological problems of the theory and practice of economic zoning.

At the same time, the first stage of the experiment showed that the method used was not quite adapted to the tasks of economic zoning. In particular, the algorithm used did not let us calculate the essential heterogeneity and incommensurability of the attributes by which the area was divided up. In preparing the initial information, therefore, we tried to make the attributes commensurable beforehand, by employing scales with equal ranges for them, taking the value of the attributes for grouping objects as equal. The drawbacks of this equating of attributes led to a considerable number of areals that contained only one or two objects of recognition (settlements).

Doubts about the correctness of the method of equating attributes used also arose at the stage of formulation of the experimental solution, because it called for supplementary weighting, for an equivalent to the choice of "standard" units of measurement along the attribute axis.

The difficulties of this equivalent weighting of the attributes of recognition can be overcome, in our view, by another approach: namely, that which we used in the second stage of dividing the territory of the central districts of

Irkutsk Region into areals characterised by similar conditions of economic development.

The exercise of delineating areals in Irkutsk Region was posed as a problem of finding the optimum of the proposed criterion of divisibility as a whole. This criterion of quality is a modification of the intersection index of the samples employed in discriminant analysis. It is essential that the "proximity" of objects for a given criterion should not depend on the units of measurement of the attributes concerned. In that way one can avoid the difficulties of equivalent weighting of attributes that arose in the first stage of the solution, and can also employ similar initial information for the second stage.

In addition, the quality criterion makes it possible to compare the results of dividing a territory into a different number of areals, and so to find their optimum spatial structure. As a result of solving the problem the statistical characteristics of the groups formed by all attributes were determined, which helped interpret the zoning scheme obtained consistently.

The series of computer calculations of the second experimental solution of the problem of defining the boundaries of territorial combinations made it possible to delineate ten groupings of objects in the central part of Irkutsk Region. The areals of the territory studied had more clearly expressed spatial parameters than in the first solution and, which is particularly important for defining their boundaries, considerably fewer groups with a minimum number of objects (settlements).

Although the new scheme had areals that represented a single settlement, their number was much reduced compared with the first solution. There were now only four such areals; and in the solution they belonged to one or other of the groups characterised by similar conditions of economic development.

The theoretical scheme of the algorithm used in the second stage of the solution thus significantly improved the potentiality of pattern recognition as a technique for generalising the sets of objects being studied, and enabled us to make a representative description of the individual features of the economic development of the areals of Irkutsk Region.

Analysis of the scheme obtained, however, indicated that even use of the new algorithm would not exclude areals representing one settlement or with complicated shapes (ribbon, sinuous, etc.). In our view this was linked with an underestimation during preparation of the initial data of the specific nature of geographic position of the objects within the boundaries of the territory being studied, which stood out by itself among the other attributes characterising their economic and geographic features.

One way of solving the problem presented was to make a certain modification of the proposed quality criterion for dividing up the area into regional combinations (areals).

We represented each object as a vector in the space $E = R^T + R^M$, in which R^T was the sub-space of the attributes describing the geographic position of an object, and R^M —the sub-space of its other economico-geographic features. The specific nature of the problem being considered thus required the criterion employed to have the following properties:

(1) invariance as regards all diagonal transformations and shifts in R^T;

(2) invariance as regards shifts and similar tensions, but non-invariance as regards other transformations in R^T;

(3) the possibility of allowing for the different significance of the various economico-geographic characteristics and geographic position, depending on the purpose of dividing the territory up.

The first property means that conclusions about the similarity of objects or the proximity of their groups do not depend on the units of measurement and points of origin of the co-ordinates on the heterogeneous axes of the characteristics. The second property is linked with the fact that the metric is given in R^T and can be measured in different units, while readings can be made from any point of origin of the co-ordinates. Note, too, that when the first two properties are met the requirement of allowing differently for geographic position and the set of other economico-geographic characteristics is satisfied.

Different variants of allowing for the geographic position of objects are now being studied, starting from the specific character of the problems of dividing up a territory. In the second stage of the solution of this problem for Irkutsk Region, when economic-geographic information already available was employed, several different initial divisions of the area were therefore tested; the final result from them proved very stable.

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